

- 1) Find the curl,  $\nabla \times \mathbf{F}$ , for the following forces: (a)  $\mathbf{F} = k\mathbf{r}$ ; (b)  $\mathbf{F} = (Ax, By^2, Cz^3)$ ; (c)  $\mathbf{F} = (Ay^2, Bx, Cz)$ , where A, B, C and k are constants.
- 2) Verify that the gravitational force  $-\frac{GMm\mathbf{r}}{r^2}$  on a point mass m at  $\mathbf{r}$ , due to a fixed point mass M at the origin, is conservative and calculate the corresponding potential energy.
- 3) A mass m is in uniform gravitational field, which exerts the usual force  $\mathbf{F} = m\mathbf{g}$  vertically down, but with g varying with time,  $g = g(t)$ . Choosing axes with y measured vertically up and defining  $U = mgy$  as usual, show that  $\mathbf{F} = -\nabla U$  as usual, but, by differentiating  $E = \frac{1}{2}mv^2 + U$  with respect to t, show that E is not conserved.
- 4) Verify the three equations:  $x = r \sin \theta \cos \phi$ ,  $y = r \sin \theta \sin \phi$ , and  $z = r \cos \theta$  that give x, y, z in terms of the spherical polar coordinates  $r, \theta, \phi$ . (b) Find expressions  $r, \theta, \phi$  in terms of x, y, z.
- 5) Consider a head-on elastic collision between two particles. Prove that the relative velocity after the collision is equal and opposite to that before. That is,  $v_1 - v_2 = -(v'_1 - v'_2)$ , where  $v_1$  and  $v_2$  are the initial velocities and  $v'_1$  and  $v'_2$  the corresponding final velocities.
- 6) A particle of mass  $m_1$  and speed  $v_1$  collides with a second particle of mass  $m_2$  at rest. If the collision is perfectly inelastic, what fraction of the kinetic energy is lost in the collision? Comment on your answer for the cases that  $m_1 \ll m_2$  and that  $m_2 \ll m_1$ .