Math 1426 **Lab 10**
Fall 2006 **Week of November 13-17**

Many students incorrectly evaluate the indeterminate forms of type 0^0, type ∞^0, and type 1^∞ as 1 because they think that "anything to the zero power is 1" and "1 to any power is 1." These rules are indeed true for powers of *numbers*. But 0^0, ∞^0, and 1^∞ are not powers of numbers but *descriptions of limits*. In this lab, we will see that these indeterminate forms can produce limits that are nonnegative real numbers or limits that are infinite.

1. We will need to use the following result from Chapter 2: if $\lim_{x\to b} f(x) = c$, then $\lim_{x\to b} e^{f(x)} = e^c$. Find the theorem that justifies this result.

2. An indeterminate form of the type 0^0 can be any positive real number. Let *a* be a positive real number. Show that $\lim_{x\to 0^+}\left[x^{(\ln a)/(1+\ln x)}\right] = a$. **To evaluate this limit:** First set $y = \left[x^{(\ln a)/(1+\ln x)}\right]$. Take the natural log of both sides, simplify, and use L'Hôpital's Rule to find the limit. Then apply #1.

3. An indeterminate form of the type ∞^0 can be any positive real number. Let *a* be a positive real number. Show that $\lim_{x\to +\infty}\left[x^{(\ln a)/(1+\ln x)}\right] = a$.

4. An indeterminate form of the type 1^∞ can be any positive real number. Let *a* be a positive real number. Show that $\lim_{x\to 0^+}(x+1)^{(\ln a)/x} = a$.

5. An indeterminate form of type 1^∞ can have an infinite limit. Show that $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{e^x} = \infty$.

6. This is an example that L'Hôpital used in his 1696 book (the first calculus textbook ever published) to illustrate the method we now call L'Hôpital's Rule. Find

$$\lim_{x\to a}\frac{\sqrt{2a^3x - x^4} - a\sqrt[3]{a^2x}}{a - \sqrt[4]{ax^3}}$$

where *a* is a positive constant.

7. Explain why L'Hôpital's rule is of no help in finding the limit below. Then compute the limit using the methods of Chapter 2.

$$\lim_{x\to +\infty}\frac{x + \sin 2x}{x}$$