Problem 1. Consider a particle of mass $m = 0.1000$ kg, connected to a fixed point by a spring of force constant $k = 10.00$ N/m. The mass is subject to a damping force

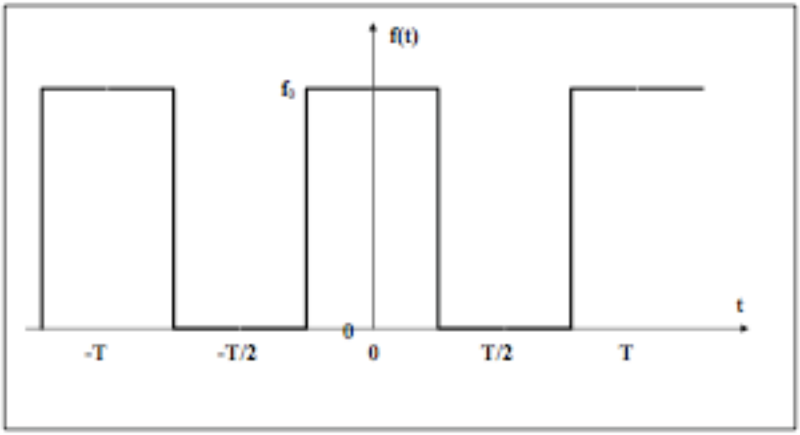
$$f_{res} = -2\beta mv,$$

where the damping force constant is given by $\beta = 1.000\ \text{sec}^{-1}$.

(a) Give the general expression for $x(t)$, the position of the particle as a function of time, with numerical values for all constants involved (to 4 significant figures).

(b) Suppose that initially (at time $t = 0$) the particle is at position $x_0 = 0$, with velocity $v_0 = 1.00$ m/s. Give the expression for the position $x(t)$ of the particle as a function of time, with numerical values for all constants involved (to 4 significant figures). Calculate the particle's velocity and position after exactly 3 seconds.

(c) Calculate the particle's total energy (kinetic plus potential) as a function of time. Evaluate the energy in Joules after exactly 3 seconds.



(d) How long (in seconds) will it take for the particle's time-averaged potential energy to become about equal to 1/2 $k_B T$, the average thermal energy per degree of freedom, at room temperature? Here T is the temperature in Kelvins, and the Boltzman constant k_B is equal to $k_B = 1.38 \times 10^{-23}$ J/K.

(e) Now suppose that the damped oscillator is driven by a driving force $f(t)$, given by the square wave of period $T = 1.000$ sec and amplitude $f_0 = 25.0$ N, as shown in the figure, with corresponding frequency $\nu = 1/T$. Calculate the Fourier coefficients a_n and b_n for this signal.

(f) Calculate the response of the damped oscillator to this driving force, and give the amplitudes, in meters, of the three largest sinusoidal components.