1) Consider a system comprising two extended bodies, which have masses M_1 and M_2 and centers of mass R_1 and R_2. Prove that the CM of whole system is at $R = (M_1R_1 + M_2R_2) / (M_1 + M_2)$.

2) A uniform thin sheet of metal is cut in the shape of a semicircle of radius R and lies in the xy plane with its center at the origin and diameter lying along the x axis. Find the position of the CM using polar coordinates.

3) A particle of mass m is moving on a frictionless horizontal table and is attached to a massless string, whose other end passes through a hole in the table, where I am holding it. Initially the particle is moving in a circle of radius r_o with angular velocity w_o, but I now pull the string down through the hole until a length r remains between the hole and the particle. What is the particle's angular velocity now?

4) A juggler is juggling a uniform rod one end of which is coated in tar and burning. He is holding the rod by the opposite end and throws it up so that, at the moment of release, it is horizontal, its CM is traveling vertically up at speed v_o and it is rotating with angular velocity w_o. To catch it, he wants to arrange that when it returns to his hand it will have made an integer number of complete rotations. What should v_o be, if the rod is to have made exactly n rotations when it returns to his hands?

5) For a system of just three particles, go through in detail the argument leading from L = Summation (on top of summation is N, on bottom is alpha = 1) of ℓ_α = Summation (N on top, alpha = 1 on bottom) r_α x p_α to L dot = Γ^{ext}, writing out all the summations explicitly.

6) Find the moment of inertia of a uniform disc of mass M and radius R rotating about its axis, by replacing the sum (I = summation (α = 1 on bottom, N on top) $m_\alpha \rho_\alpha^2$) by the appropriate integral and doing the integral in polar coordinates.

7) Show that the moment of inertia of a uniform solid sphere rotating about a diameter is 2/5 MR^2. The sum (I = summation (α = 1 on bottom, N on top) $m_\alpha \rho_\alpha^2$) must be replaced by an integral, which is easiest in spherical polar coordinates, with the axis of rotation taken to be the z axis. The element of volume is dV = r^2 dr sinθ dθ dϕ.

8) Consider a uniform solid disk of mass M and radius R, rolling without slipping down an incline which is at an angle γ to the horizontal. The instantaneous point of contact between the disk and the incline is called P. (a) Draw a free body diagram, showing all forces on the disk. (b) Find the linear acceleration v dot of the disk by applying the result L dot = Γ^{ext} for rotation about P. (c) Derive the same result by applying L dot = Γ^{ext} to the rotation about the CM.

9) A system consists of N masses m_α at positions r_α relative to a fixed origin O. Let r'_α denote the position of m_α relative to the CM; that is $r'_\alpha = r_\alpha - R$. (a) Make a sketch to illustrate this last equation. (b) Prove the useful relation that $\Sigma\, m_\alpha r'_\alpha = 0$. Can you explain why this relation is nearly obvious? (c) Use this relation to prove the result (d/dt) L (about CM) = Γ^{ext} (about CM) that the rate of change of the angular momentum about the CM is equal to the total external torque about the CM.