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Question 1

Consider a following simple moral hazard model. There is a risk-neutral principal and a risk-averse agent. The agent can either work hard (a_H) or shirk (a_L). There are two possible outcomes: high output ($y = 150$) or low output ($y = 100$).

The output depends on the agent's effort as follows: if the agent works hard (a_H), the high output occurs with probability 0.5 and the low output occurs with probabilit 0.5. If the agent shirks (a_L), the high output occurs with probability 0.4 and the low output occurs with probability 0.6. To summarize, the probability distribution of output is as follows:

	$y = 150$	$y = 100$
a_H	0.5	0.5
a_L	0.4	0.6

The agent is risk-averse, and has the utility function $U(w, a) = w - C_a$ where w is the wage and C_a is the cost of effort. If the agent works hard, $C_a = 10$. If the agent shirks, $C_a = 0$. The agent's alternative utility is 50.

The agent's effort is not observable to the principal, but the output is observable. Thus, the wage contract is as follows: If the output is low, the agent receives $w = s$. If the output is high, the agent receives, $w = s + b$.

Assume that if possible, the principal wants to set $b = 0$, for example, to avoid measuring output.

a) If $s = 10$ and $b = 5$, will the agent work hard?

b) In general, for arbitrary s and b, when would the agent work hard? (i.e. write down the incentive constraint)

c) Given that the agent works hard, when would the agent accept the contract (s, b) ? (i.e. write down the participation constraint)

d) Carefully explain that both the participation and the incentive constraint must hold with equality.

e) Compute the optimal s and b. And compute the principal's expected profit, $E[y - w]$.

f) Suppose that the principal wants to hire the agent, but does not want the agent to work hard. In such a case, what is the optimal contract (s, b)? And compute the principal's expected profit.

g) Show that it is optimal for the principal NOT to induce hard work from the agent. Explain why.

h) Now suppose that the probability distribution of output is as follows:

	$y = 150$	$y = 100$
a_H	0.8	0.2
a_L	0.2	0.8

(i) Show that it now optimal for the principal to induce hard work from the agent.

(ii) Explain why the answer in [g] has changed now.

Question 2 **[The numbers will not be pretty in this question. Try to get a good calculator :)]**

In the same set-up as question [2], now suppose that there are three possible outcomes: $y = 5000$, $y = 4500$, and $y = 4000$. The probability distribution of output is as follows:

	$y = 5000$	$y = 4500$	$y = 4000$
a_H	0.5	0.3	0.2
a_L	0.2	0.3	0.5

That is, if the agent works hard, $y = 5000$ with probability a half, and $y = 4500$ with probability 0.3, and $y = 4000$ with probability 0.2, and so on.

Also, now the agent is risk-averse. So the agent's utility function is $U(w, a) = \sqrt{w} - C_a$, where C_a is defined as in question 1.

The wage contract is now as follows: (w_L, w_M, w_H) where w_L is the wage when $y = 4000$, w_M is the wage payment when $y = 4500$, and w_H is the wage payment when $y = 5000$.

Suppose that the principal wants to induce high effort.

a) For a given contract (w_L, w_M, w_H), specify the incentive constraint.

b) Specify the participation constraint.

c) Specify the principal's optimization problem.

d) Compute the optimal contract (w_L^*, w_M^*, w_H^*). [Hint, define $v_L = \sqrt{w_L}$, $v_M = \sqrt{w_M}$, $v_H = \sqrt{w_H}$ so that $w_L = v_L^2$, $w_M = v_M^2$, $w_H = v_H^2$. Then, solve for the optimal (v_L^*, v_M^*, v_H^*).]

Question 3 **(Use Question3.xls to answer this question.)**

Paul is starting a new job with SuperKleen janitorial services. Provided he is not fired for shirking, Paul will stay with SuperKleen for twenty years. His productivity (assuming he does not shirk) and value of leisure (or alternative wage) over this period are given by the first two columns of the attached spreadsheet.

SuperKleen is deciding between two wage lifetime profiles it might offer to Paul. These are labelled as wage profiles A and B in the attached spreadsheet. The discount rate is zero (or discount factor is one).

a) What is the present value of Paul's lifetime productivity? What is the present value of the wages he receives from SuperKleen under profile A? Under profile B?

What words might you use to describe the compensation policy embodied in each of these two schemes?

b) When is Paul's efficient retirement date? If he is paid according to profile A will he retire voluntarily at that date? What if he is paid according to profile B?

c) For each year Paul works at SuperKleen, use a spreadsheet to calculate the remaining present value of Paul's productivity at SuperKleen, i.e. what's the present value of output he has yet to produce (including the current year) at each date of his career. (Hint: start at Paul's last year with SuperKleen and work backwards).

d) Do the same thing as in part (c), but for Paul's wages (in separate columns for the A and B wage profiles). In other words, at each date of his career, find the present value of the wages Paul has yet to receive from SuperKleen (if he stays on the job and doesn't shirk). Do this for the remaining present value of leisure too (this is what Paul gets if he stops working altogether at any particular date).

e) If Paul shirks (for example he sleeps on the job) and is caught, he is fired. One option he has after being fired is to take an alternative job where (because the workers are easily monitored) shirking is not an issue. In this job he produces 50 units of output less each year than at SuperKleen. Add columns to your spreadsheet that calculate Paul's productivity each year on the alternative job, and his remaining present value of productivity on that job. Assume that, if he takes this job, the firm offering it just breaks even, i.e. over the course of his career there, he receives a wage profile equal in present value to his remaining productivity there.

f) Now consider Paul's incentives to shirk in his job at SuperKleen. If he shirks in any given year, he experiences a gain in on-the-job leisure that year that is worth 50 to him. In that event, he is detected with probability p=.25 (set up your spreadsheet so you can input alternative values of p, and of the value of the extra leisure he gets from shirking).

If he is detected, Paul is fired and takes his next best alternative (which is either the alternative job or leisure, i.e. early retirement, whichever is higher) for the rest of his career. So, overall, Paul's net gain to cheating consists of:

- The extra on-the-job leisure of 50 (which he gets for sure)
- A (1-p) chance of losing nothing
- A p chance of the difference between his remaining present value of wages at SuperKleen and the remaining present value of his next best alternative to working at SuperKleen.

In your spreadsheet, calculate Paul's net gain to shirking in each year of his career under wage profile A, and under wage profile B. (Hint: use excel's "max" function to pick the higher of two arguments).

g) What are Paul's net gains from shirking in his 17th year at SuperKleen under wage profile A? Under profile B? Overall, which profile gives him the greater incentive to shirk? Comment on which stages of his work life where Paul is most tempted to shirk under each profile, and explain.

h) Increase the on-the-job leisure Paul gets from shirking from 50 to 100. Does wage profile B still prevent him from shirking throughout his career? Now put this value back to 50 and reduce the firm's detection probability to .1. Now does wage profile B prevent shirking? In each of the above cases, if profile B does not prevent shirking, how would you modify it to prevent shirking?

Question 4

Consider a tournament contract with two agents. An agent with higher outputs wins the tournament and receives the wage 150. The agent with lower output receives the wage 100.

The output of agent 1 is determined by $q_1 = 2a_1 + 0.5\epsilon$ where a_1 is the agent 1's effort and ϵ is a random shock. The output of agent 2 is determined by $q_2 = a_2 - 0.5\epsilon$ where a_2 is the agent 2's effort. Note that agent 1 has higher ability in a sense that when both agent exerts the same amount effort, agent 1 is more likely to have higher output.

The random shock is distributed between -1 and 1. Assume that the probability that $\epsilon > X$ is given by

$$\Pr(\epsilon > X) = \frac{3}{4} - \frac{1}{2}X - \frac{1}{4}X^2$$

Each agent's utility function is $U(w, a) = w - a^2/2$, where w is the wage and $a^2/2$ is the cost of effort.

a) For a given effort (a_1, a_2), what is the probability that agent 1 will win the tournament? What is the probability that agent 2 will win the tournament?

b) For the given contract, specify each agent's expected utility.

c) Express each agent's incentive constraint.

d) For the given contract, compute the equilibrium effort level of each agent.

e) Suppose that the agent 1 has the same ability as agent 2. That is, $q_1 = a_1 + 0.5\epsilon$. Compute the equilibrium effort level of each agent.

f) Compare your answer in d) and e). When do the agents work harder? Explain the intuition.

Question 5

Suppose that you want to empirically test the implications of the optimal piece-rate model. You have data on individual workers' performance (y_i), wage payment (w_i), wealth (W_i), and job tenure (t_i).

You can estimate the following wage equation:

$$w_i = \alpha_0 + \alpha_1 y_i + \alpha_2 W_i + \alpha_3 t_i + \epsilon_i$$

where ϵ_i is a random noise.

a) Among the coefficients you can estimate (i.e. $\alpha_0, \alpha_1, \alpha_2, \alpha_3$), which coefficient corresponds to the piece-rate?

b) Suppose that the agent's risk-aversion decreases with his/her wealth. The optimal piece-rate model predicts that the piece-rate should decrease in the agent's risk-aversion. In order to test this prediction, how would you modify the wage equation above?

c) Some argue that as a worker's tenure increases, the principal should provide stronger piece-rate. Explain why?

d) If you want to test (c), how would you modify the wage equation?

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