7.2.14 Show that ($a > 0$)

(a) $$\int_{-\infty}^{\infty} \frac{\cos x}{x^2 + a^2}\,dx = \frac{\pi}{a}e^{-a}.$$

How is the right side modified if $\cos x$ is replaced by $\cos kx$?

(b) $$\int_{-\infty}^{\infty} \frac{x \sin x}{x^2 + a^2}\,dx = \pi e^{-a}.$$

How is the right side modified if $\sin x$ is replaced by $\sin kx$?

These integrals may also be interpreted as Fourier cosine and sine transforms—Chapter 15.