

7. Arfken, p. 464, 5<sup>th</sup> Ed., Prob. 7.2.7 (p. 476, Prob. 7.1.7, 6<sup>th</sup> Ed.). It is probably best to first note that all the possible choices of integrals are the same, due to the transformation properties of  $\cos \theta \rightarrow -\cos \theta \rightarrow \sin \theta \rightarrow -\sin \theta$ , if the integral is over all angles  $0 \leq \theta < 2\pi$ . Given this fact, you will need to do one representative integral, such as  $\int_0^{2\pi} \frac{d\theta}{a + b \cos \theta}$ , for  $a > |b|$ . It is probably best to approach this integral by making the integration variable substitution  $z = e^{i\theta}$ ,  $dz = ie^{i\theta} d\theta = iz d\theta$ ,  $d\theta = \frac{dz}{iz}$ , where the integral in the complex plane is simply a closed loop contour,  $C$ , of radius one,  $|z| = 1$ , about the origin. In addition, replace the cosine function with  $\cos \theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta}) = \frac{1}{2}\left(z + \frac{1}{z}\right)$ . Given the equivalent closed loop integral in the complex plane, you will need to use the residue theorem in order to simply calculate the integral,  $\oint_C f(z) dz = 2\pi i$  (sum of residues). Recall that the residue is the coefficient,  $a_0$ , of