2. **Arfken, p. 342, 5th Ed. (p. 359, 6th Ed.), Prob. 5.6.7.** Use the General Vector Taylor Series Expansion For a General Function, $\Phi(\mathbf{r}) = \Phi(x, y, z)$, Of a Three-Dimensional Vector Coordinate, Expressed In Cartesian Coordinates, which is Expanded About the Origin, $\mathbf{r} = \mathbf{0}$ Or $x = y = z = 0$, Where

$$\Phi(\mathbf{r}') = \Phi(x', y', z') = \sum_{n=0}^{\infty} \frac{1}{n!} (\mathbf{r}' \cdot \nabla)^n \, \Phi(\mathbf{0}) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(x' \frac{\partial}{\partial x} + y' \frac{\partial}{\partial y} + z' \frac{\partial}{\partial z} \right)^n \Phi(0,0,0),$$

With the Notation that $\frac{\partial^n}{\partial x^n} \Phi(0,0,0), \frac{\partial^n}{\partial y^n} \Phi(0,0,0), \frac{\partial^n}{\partial z^n} \Phi(0,0,0)$, Are the nth Order Derivatives Of the Function, $\Phi(x, y, z)$, Evaluated At the Origin, $(x, y, z) = (0,0,0)$, Which Are Constants. In Addition, In This Notation, the Laplacian Of the Function Evaluated At the Origin Is Denoted As

$$\nabla^2 \Phi(\mathbf{r})\Big|_{\mathbf{r}=\mathbf{0}} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Phi(x, y, z)\Bigg|_{x=y=z=0}$$

$$= \frac{\partial^2}{\partial x^2} \Phi(0,0,0) + \frac{\partial^2}{\partial y^2} \Phi(0,0,0) + \frac{\partial^2}{\partial z^2} \Phi(0,0,0)$$

. Expand the Function,