

Consider $C[0, 1]$, the space of real valued continuous functions defined on the unit interval $[0, 1]$. Let

$$K = C^1[0, 1] \cap \{f : \int_0^1 f'^2 \leq 1, \|f\|_\infty \leq 1\}$$

Note that $C^1[0, 1] \subset C[0, 1]$, and $K \subset C[0, 1]$. Show that K is compact in C .

I am assuming compactness here refers to the sequential compactness. This seems to make the most sense. Since this problem is an analysis problem, please be sure to be rigorous, and include as much detail as possible so that I can understand. Please also state if you are making use of some fact or theorem. Thanks!