o make these computations simpler, note that

$$\sum_{i=1}^{n}(X_{1i}-\bar{X}_1)^2=\sum_{i=1}^{n}X_{1i}^2-\frac{\left(\sum_{i=1}^{n}X_{1i}\right)^2}{n}$$

$$\sum_{i=1}^{n}(X_{2i}-\bar{X}_2)^2=\sum_{i=1}^{n}X_{2i}^2-\frac{\left(\sum_{i=1}^{n}X_{2i}\right)^2}{n}$$

$$\sum_{i=1}^{n}(X_{1i}-\bar{X}_1)(Y_i-\bar{Y})=\sum_{i=1}^{n}X_{1i}Y_i-\frac{\left(\sum_{i=1}^{n}X_{1i}\right)\left(\sum_{i=1}^{n}Y_i\right)}{n}$$

$$\sum_{i=1}^{n}(X_{2i}-\bar{X}_2)(Y_i-\bar{Y})=\sum_{i=1}^{n}X_{2i}Y_i-\frac{\left(\sum_{i=1}^{n}X_{2i}\right)\left(\sum_{i=1}^{n}Y_i\right)}{n}$$

$$\sum_{i=1}^{n}(X_{1i}-\bar{X}_1)(X_{2i}-\bar{X}_2)=\sum_{i=1}^{n}X_{1i}X_{2i}-\frac{\left(\sum_{i=1}^{n}X_{1i}\right)\left(\sum_{i=1}^{n}X_{2i}\right)}{n}$$

The following example illustrates how least-squares estimates of A, B_1, normal) equations:

$$\sum_{i=1}^{n} Y_i = na + b_1 \sum_{i=1}^{n} X_{1i} + b_2 \sum_{i=1}^{n} X_{2i}$$

$$\sum_{i=1}^{n} X_{1i} Y_i = a \sum_{i=1}^{n} X_{1i} + b_1 \sum_{i=1}^{n} X_{1i}^2 + b_2 \sum_{i=1}^{n} X_{1i} X_{2i}$$

$$\sum_{i=1}^{n} X_{2i} Y_i = a \sum_{i=1}^{n} X_{2i} + b_1 \sum_{i=1}^{n} X_{1i} X_{2i} + b_2 \sum_{i=1}^{n} X_{2i}^2. \tag{12.4}$$

Solving these equations for a, b_1, and b_2, we obtain the following results:

$$b_1 = \frac{\sum_{i=1}^{n} (X_{2i} - \bar{X}_2)^2 \sum_{i=1}^{n} (X_{1i} - \bar{X}_1)(Y_i - \bar{Y}) - \sum_{i=1}^{n} (X_{1i} - \bar{X}_1)(X_{2i} - \bar{X}_2) \sum_{i=1}^{n} (X_{2i} - \bar{X}_2)(Y_i - \bar{Y})}{\sum_{i=1}^{n} (X_{1i} - \bar{X}_1)^2 \sum_{i=1}^{n} (X_{2i} - \bar{X}_2)^2 - \left[\sum_{i=1}^{n} (X_{1i} - \bar{X}_1)(X_{2i} - \bar{X}_2)\right]^2}$$

$$b_2 = \frac{\sum_{i=1}^{n} (X_{1i} - \bar{X}_1)^2 \sum_{i=1}^{n} (X_{2i} - \bar{X}_2)(Y_i - \bar{Y}) - \sum_{i=1}^{n} (X_{1i} - \bar{X}_1)(X_{2i} - \bar{X}_2) \sum_{i=1}^{n} (X_{1i} - \bar{X}_1)(Y_i - \bar{Y})}{\sum_{i=1}^{n} (X_{1i} - \bar{X}_1)^2 \sum_{i=1}^{n} (X_{2i} - \bar{X}_2)^2 - \left[\sum_{i=1}^{n} (X_{1i} - \bar{X}_1)(X_{2i} - \bar{X}_2)\right]^2}$$

$$a = \bar{Y} - b_1 \bar{X}_1 - b_2 \bar{X}_2. \tag{12.5}$$