Problem 1.33 Test Stokes' theorem for the function $\mathbf{v} = (xy)\,\hat{\mathbf{x}} + (2yz)\,\hat{\mathbf{y}} + (3zx)\,\hat{\mathbf{z}}$, using the triangular shaded area of Fig. 1.34.

Fundamental theorem of curls, $\int_S (\nabla \times \vec{v}) \cdot da = \oint \vec{v} \cdot d\vec{l}$,
calculate the area integral over the curl of the vector field on the triangle area, then calculate the line integral of the vector field around the perimeter of the triangle (there are 3 integrals), compare the two to show that they are the same, and give the result.

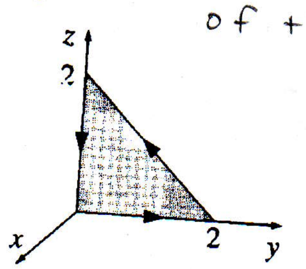

Figure 1.34

Problem 1.39 Compute the divergence of the function (spherical coordinates)

$$\mathbf{v} = (r\cos\theta)\,\hat{\mathbf{r}} + (r\sin\theta)\,\hat{\boldsymbol{\theta}} + (r\sin\theta\cos\phi)\,\hat{\boldsymbol{\phi}}.$$

Problem 1.42

(a) Find the divergence of the function (cylindrical coordinates)

Find the divergence of the vector field, $\nabla \cdot \vec{v}$

$$\mathbf{v} = s(2 + \sin^2\phi)\,\hat{\mathbf{s}} + s\sin\phi\cos\phi\,\hat{\boldsymbol{\phi}} + 3z\,\hat{\mathbf{z}}.$$

Problem 1.43 Evaluate the following integrals: Use the delta function

(a) $\int_2^6 (3x^2 - 2x - 1)\,\delta(x-3)\,dx$.

(b) $\int_0^5 \cos x\,\delta(x-\pi)\,dx$.

(c) $\int_0^3 x^3\delta(x+1)\,dx$.

(d) $\int_{-\infty}^{\infty} \ln(x+3)\,\delta(x+2)\,dx$.