Problem 1.25 Calculate the Laplacian of the following functions:

(a) $T_a = x^2 + 2xy + 3z + 4$.

(b) $T_b = \sin x \sin y \sin z$.

(c) $T_c = e^{-5x} \sin 4y \cos 3z$.

(d) $\mathbf{v} = x^2\,\hat{\mathbf{x}} + 3xz^2\,\hat{\mathbf{y}} - 2xz\,\hat{\mathbf{z}}$.

Cartesian coordinates of the scalar, $\nabla^2 T$, and vector, $\nabla^2 v$, fields given

Problem 1.28 Calculate the line integral of the function $\mathbf{v} = x^2\,\hat{\mathbf{x}} + 2yz\,\hat{\mathbf{y}} + y^2\,\hat{\mathbf{z}}$ from the origin to the point (1,1,1) by two different routes:

(a) $(0,0,0) \to (1,0,0) \to (1,1,0) \to (1,1,1)$;

(b) $(0,0,0) \to (0,0,1) \to (0,1,1) \to (1,1,1)$;

• Also given in results state if they support or not that a potential function, Φ, can be found that generates the vector function, where $\vec{v} = -\nabla\Phi$. If it is supportive, find the potential Φ

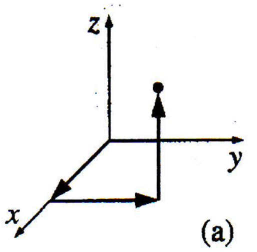
Problem 1.31 Check the fundamental theorem for gradients, using $T = x^2 + 4xy + 2yz^3$, the points $\mathbf{a} = (0,0,0)$, $\mathbf{b} = (1,1,1)$, and the one paths in Fig. 1.28:

(a) $(0,0,0) \to (1,0,0) \to (1,1,0) \to (1,1,1)$;

• Use the fundamental theorem of gradients, $\int_{a\,P}^{b} (\nabla T)\cdot d\vec{l} = T(\vec{b}) - T(\vec{a})$

• Calculate the line integrals over the gradient along the path, then calculate the difference of the scalar function, and show that they are the same, and give the result.

Fig 1.28



(a)

Problem 1.32 Test the divergence theorem for the function $\mathbf{v} = (xy)\,\hat{\mathbf{x}} + (2yz)\,\hat{\mathbf{y}} + (3zx)\,\hat{\mathbf{z}}$. Take as your volume the cube shown in Fig. 1.30, with sides of length 2.

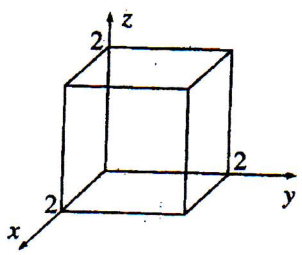


Figure 1.30

Using the fundamental theorem of divergences, $\int_V (\nabla\cdot\vec{v})\,d\tau = \oint_S \vec{v}\cdot d\vec{a}$, calculate the volume integral over the divergences of the vector field, then calculate the closed surface integral over the vector field dotted into the surface area element, for all six faces of the cube, and show that the results are the same, and give the result.