Preparing for Trigonometry
TAH Assignment Number 3

Some Algebra Review

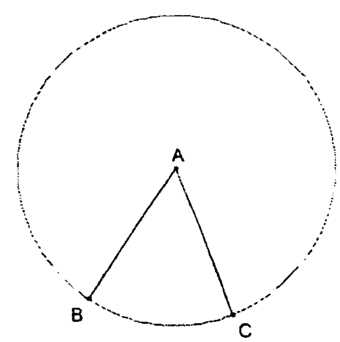
1. Write the equation of: (a) a circle centered at (3,4) with radius 5;
   (b) a line containing (-1,3) and (5,18);
   (c) the locus of points equidistant from (-5,4) and (3,16)

2. Find the points of intersection of $x^2 + y^2 = 1$ *and* $y = x\sqrt{3}$

3. Simplify $\dfrac{x^3 - y^3}{x^2 - y^2}$

4. Simplify (a) $\dfrac{6}{\sqrt{3}} + \sqrt{12}$ (b) $\dfrac{6}{\sqrt{6}}$ (c) $\dfrac{a}{\sqrt{a}}$

Some Geometry Review



Refer to circle A at right, which has radius 12.

1. Find the circumference.

2. If $\angle A = 30°$, find the length of arc BC

3. If $\angle A = 80°$, find the length of arc BC

4. If $\angle A = 120°$, find the length of arc BC

5. If arc BC has length 3π, find $\angle A$

6. If arc BC has length 16π, find $\angle A$

Some function stuff

From the text, do these problems:

pages 136-137, problems 15, 33, 35
page 143, problems 1, 3, 7
pages 149-150, problems 1, 11, 21

Some problems

From the text, do these problems:

page 207, problem 32
page 162, problem 9

Use your graphing calculator

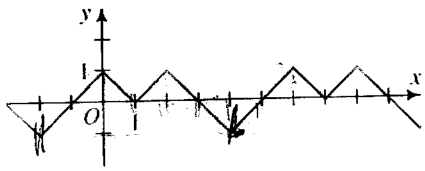
From the text, do these problems:

page 162, problem 15

WRITTEN EXERCISES

In Exercises 1–4, the graph of a function f is given. Tell whether f appears to be periodic. If so, give its fundamental period and its amplitude, and then find $f(1000)$ and $f(-1000)$.

A **1.**



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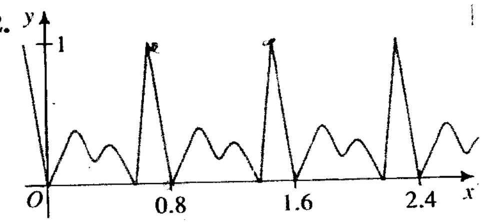

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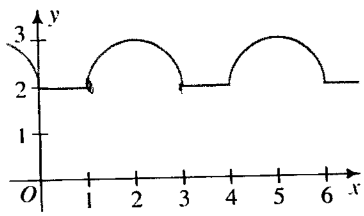

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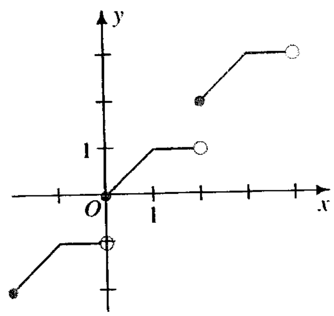
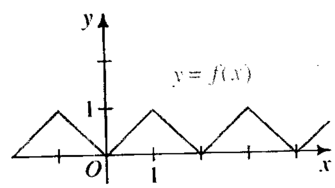


5. Use the graph of $y = f(x)$, shown at the right, to sketch the graph of each of the following.

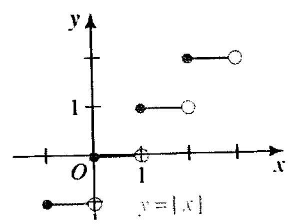


a. $y = 2f(x)$ **b.** $y = -\frac{1}{2}f(x)$

c. $y = f(-2x)$ **d.** $y = f\left(\frac{1}{2}x\right)$

e. $y = f\left(x - \frac{1}{2}\right)$ **f.** $y = f(-x) + 1$

6. The greatest integer function $y = \lfloor x \rfloor$ gives the greatest integer less than or equal to x. Thus, $\lfloor 2.1 \rfloor = 2$ and $\lfloor -3.1 \rfloor = -4$. Use the graph of this function, shown at the right, to sketch the graph of each of the following.



a. $y = \frac{1}{2}\lfloor x \rfloor$ **b.** $y = -2\lfloor x \rfloor$

c. $y = \left\lfloor -\frac{1}{2}x \right\rfloor$ **d.** $y = \lfloor 2x \rfloor$

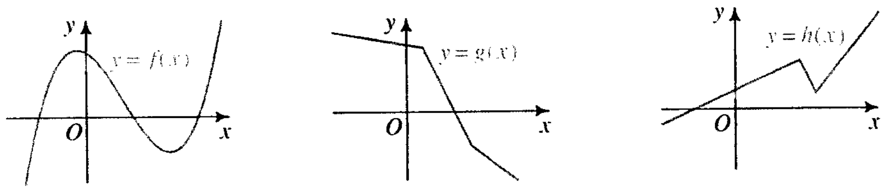
e. $y = \lfloor x - 1 \rfloor$ **f.** $y = 2\lfloor x \rfloor + 1$

7. Sketch the graph of each of the following.

a. $y + 2 = |x|$ **b.** $y = |x - 3|$ **c.** $y - 4 = |x + 5|$

d. $y = 2|x + 1|$ **e.** $y + 1 = -|x|$ **f.** $y - 3 = |2x|$

7. The graphs of f, g, and h are shown below. Which functions are one-to-one? Which functions have inverses?

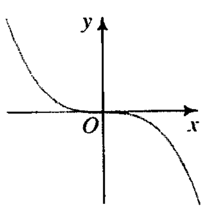


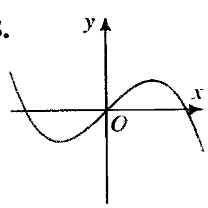
8. Which of the following functions have inverses?
   a. $f(x) = |x|$
   b. $f(x) = x^3$
   c. $f(x) = x^4$
   d. $f(x) = x^4, x \leq 0$
9. On the dial or the buttons of a telephone, a telephone function T pairs letters of the alphabet with the digits 2–9. For example, $T(\text{A}) = 2$ and $T(\text{D}) = 3$. Does T have an inverse? Explain.
10. Explain how the vertical-line test (given on page 121) can be used to justify the horizontal-line test (given on page 148).
11. Explain why the domain of a one-to-one function f is the range of f^{-1} and why the domain of f^{-1} is the range of f.

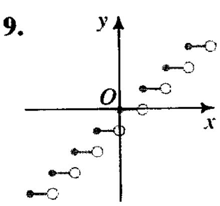
WRITTEN EXERCISES

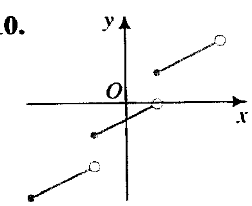
A 1. Suppose a function f has an inverse. If $f(2) = 6$ and $f(3) = 7$, find:
   a. $f^{-1}(6)$
   b. $f^{-1}(f(3))$
   c. $f(f^{-1}(7))$
2. Suppose a function f has an inverse. If $f(0) = -1$ and $f(-1) = 2$, find:
   a. $f^{-1}(-1)$
   b. $f^{-1}(f(0))$
   c. $f(f^{-1}(2))$
3. If $g(3) = 5$ and $g(-1) = 5$, explain why g has no inverse.
4. Explain why $f(x) = x^3 + x^2$ has no inverse.
5. Let $h(x) = 4x - 3$.
   a. Sketch the graphs of h and h^{-1}.
   b. Find a rule for $h^{-1}(x)$.
6. Let $L(x) = \frac{1}{2}x - 4$.
   a. Sketch the graphs of L and L^{-1}.
   b. Find a rule for $L^{-1}(x)$.

In Exercises 7–10, the graph of a function is given. State whether the function has an inverse.

7. 

8. 

9. 

10. 

State whether the function f has an inverse. If f^{-1} exists, find a rule for $f^{-1}(x)$ and show that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$.

11. $f(x) = 3x - 5$

12. $f(x) = |x| - 2$

13. $f(x) = \sqrt[4]{x}$

14. $f(x) = \frac{1}{x}$

15. $f(x) = \frac{1}{x^2}$

16. $f(x) = \sqrt{5 - x}$

17. $f(x) = \sqrt{4 - x^2}$

18. $f(x) = \sqrt{5 - x^2}$

19. $f(x) = \sqrt[3]{1 + x^3}$

Sketch the graphs of g and g^{-1}. Then find a rule for $g^{-1}(x)$.

B **20.** $g(x) = x^2 + 2,\ x \geq 0$

21. $g(x) = 9 - x^2,\ x \leq 0$

22. $g(x) = (x - 1)^2 + 1,\ x \leq 1$

23. $g(x) = (x - 4)^2 - 1,\ x \geq 4$

In Exercises 24–26, show that $h^{-1}(x) = h(x)$. Then sketch the graph of h. You may wish to use a computer or graphing calculator.

24. $h(x) = \sqrt[3]{1 - x^3}$

25. $h(x) = \frac{x}{x - 1}$

26. $h(x) = \sqrt{1 - x^2},\ x \geq 0$

27. a. Using the results of Exercises 24–26, state how the graph of h is related to the line $y = x$ when $h^{-1}(x) = h(x)$.

b. Find a function h, different from those in Exercises 24–26, such that $h^{-1}(x) = h(x)$.

28. Refer to the definition of an increasing function given on page 138.

a. Explain why an increasing function must have an inverse.

b. Suppose f is an increasing function. Is f^{-1} also an increasing function? Explain your answer and support it with at least two examples.

C **29.** Which statement below is true? Prove it.

(i) $(f \circ g)^{-1} = f^{-1} \circ g^{-1}$ (ii) $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

30. If f is a linear function such that $f(x + 2) - f(x) = 6$, find the value of $f^{-1}(x + 2) - f^{-1}(x)$.

31. Suppose a, b, and c are constants such that $a \neq 0$. Let $P(x) = ax^2 + bx + c$ for $x \leq -\frac{b}{2a}$. Find a rule for $P^{-1}(x)$.

CALCULATOR EXERCISES

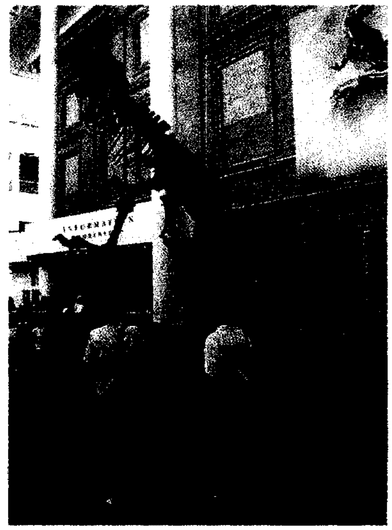
In the next chapter we will define the functions $f(x) = e^x$ and $g(x) = \ln x$. You can use a calculator to learn something about these functions.

1. Enter any number. Press the $\boxed{e^x}$ and $\boxed{\ln x}$ keys alternately several times. What do you notice? Repeat this process for several other numbers. How would you describe the relationship between $f(x) = e^x$ and $g(x) = \ln x$?
2. By entering various numbers, determine whether $f(x) = e^x$ is defined for all real numbers.
3. By experimenting, determine the domain of $g(x) = \ln x$.

31. **a.** If $b^m > b^n$ and $b > 1$, what can you say about m and n?
b. If $b^m > b^n$ and $0 < b < 1$, what can you say about m and n?

32. Solve: **a.** $8^x > 8^{7-x}$ **b.** $0.6^{5x} > 0.6^{x/2}$ **c.** $e^{3x} < e^x$ **d.** $\left(\frac{1}{2}\right)^x > 2^{6-x}$

33. **Archaeology** All living organisms contain a small amount of carbon 14, denoted C^{14}, a radioactive isotope. When an organism dies, the amount of C^{14} present decays exponentially. By measuring the radioactivity $N(t)$ of, say, an ancient skeleton of an animal and by comparing that radioactivity with the radioactivity N_0 of living animals, archaeologists can tell approximately when the animal died.



a. Given that the half-life of C^{14} is about 5700 years, write an equation relating $N(t)$, N_0, and the time t since the animal's death.
b. Suppose it is found that, for a certain animal, $N(t) = \frac{1}{10}N_0$. To the nearest 100 years, how long ago did the animal die?

34. **Archaeology** An archaeologist unearths a piece of wood that may have come from the Hanging Gardens of Babylon, about 600 B.C. The amount of radioactive C^{14} in the wood is $N(t) = 0.8N_0$. Is it possible that the wood could be from the Hanging Gardens? (The half-life of C^{14} is about 5700 years.)

35. Prove: $\log_b c = \dfrac{1}{\log_c b}$

36. Prove: $(\log_a b)(\log_b c) = \log_a c$

Evaluate each expression. (Use the results of Exercises 35 and 36.)

37. $\log_3 2 \cdot \log_2 27$ 38. $\log_{25} 8 \cdot \log_8 5$ 39. $\dfrac{1}{\log_2 6} + \dfrac{1}{\log_3 6}$ 40. $\dfrac{1}{\log_4 6} + \dfrac{1}{\log_9 6}$

For Exercises 41 and 42, use a computer or a graphing calculator to solve each inequality. Give answers to the nearest hundredth.

41. On a single set of axes, graph $y = \log_2 (x - 1)$ and $y = \log_3 x$. (See Class Exercise 9 on page 205.) Use your graph to solve $\log_2 (x - 1) > \log_3 x$.

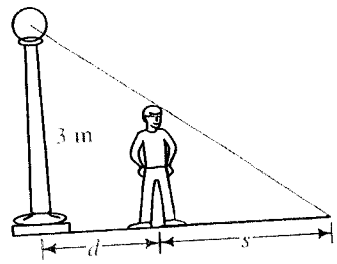
42. Solve each inequality using the method suggested in Exercise 41.
a. $e^x < \ln (x + 5)$ **b.** $2^x \leq \log_5 x$ **c.** $\log 20x > 2^{-x}$ **d.** $\log x \geq \log_4 x^2$

43. **Oceanography** After passing through a material t centimeters thick, the intensity $I(t)$ of a light beam is given by $I(t) = (4^{-ct})I_0$, where I_0 is the initial intensity and c is a constant called the absorption factor. Ocean water absorbs light with an absorption factor of $c = 0.0101$. At what depth will a beam of light be reduced to 50% of its initial intensity? 2% of its initial intensity?

C 44. Prove: $a^{\log b} = b^{\log a}$

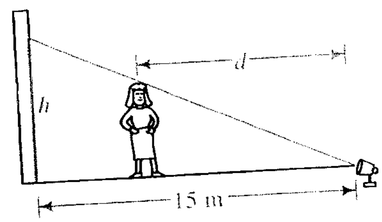
45. Prove: $\dfrac{1}{\log_a ab} + \dfrac{1}{\log_b ab} = 1$

9. A light 3 m above the ground causes a boy 1.8 m tall to cast a shadow s meters long measured along the ground, as shown at the right. Express s as a function of d, the boy's distance in meters from the light.



Ex. 9

10. When a girl 1.75 m tall stands between a wall and a light on the ground 15 m away, she casts a shadow h meters high on the wall, as shown at the right. Express h as a function of d, the girl's distance in meters from the light.



Ex. 10

11. A box with a square base has a surface area (including the top) of 3 m^2. Express the volume V of the box as a function of the width w of the base.

12. A box with a square base and no top has a volume of 6 m^3. Express the total surface area A of the box as a function of the width w of the base.

13. A stone is thrown into a lake, and t seconds after the splash the diameter of the circle of ripples is t meters.
    a. Express the circumference C of this circle as a function of t.
    b. Express the area A of this circle as a function of t.

14. A balloon is inflated in such a way that its volume increases at a rate of 20 cm^3/s.
    a. If the volume of the balloon was 100 cm^3 when the process of inflation began, what will the volume be after t seconds of inflation?
    b. Assuming that the balloon is spherical while it is being inflated, express the radius r of the balloon as a function of t.

Part (b) of Exercises 15 and 16 requires the use of a computer or graphing calculator.

B 15. **Manufacturing** A box with a square base and no top has volume 8 m^3. The material for the base costs \$8 per square meter, and the material for the sides costs \$6 per square meter.
    a. Express the cost C of the materials used to make the box as a function of the width w of the base.
    b. Use a computer or graphing calculator to find the minimum cost.

16. **Manufacturing** A cylindrical can has a volume of 400π cm^3. The material for the top and bottom costs 2¢ per square centimeter. The material for the vertical surface costs 1¢ per square centimeter.
    a. Express the cost C of the materials used to make the can as a function of the radius r.
    b. Use a computer or graphing calculator to find the minimum cost.