

Let l, m, n be distinct lines and P, Q, R be distinct points. Prove the following:

- (a) $\sigma_l\sigma_m = \sigma_m\sigma_l$ if and only if $l \perp m$.
- (b) $\sigma_P\sigma_m = \sigma_m\sigma_P$ if and only if $P \in m$.
- (c) $\sigma_l\sigma_m\sigma_n = \sigma_n\sigma_m\sigma_l$ if and only if l, m, n are either concurrent or parallel.
- (d) $\sigma_P\sigma_Q = \sigma_Q\sigma_R$ if and only if Q is the midpoint of \overline{PR} .
- (e) $\sigma_P\sigma_l = \sigma_l\sigma_Q$ if and only if l is the perpendicular bisector of \overline{PQ} .