INTEGER PROGRAMMING

- In Linear Programming formulations we have assumed that the decision variables are *divisible* (i.e., fractional values are acceptable)
- In many decision making situations fractional values for the decision variables are perfectly acceptable. (e.g., number of Tons of chemicals to be produced).
- In many decision making situations fractional values for decision variables are not always meaningful. For examples
  - Number of employees to be hired
  - Number of houses to be built
- Thus, in certain formulations, it becomes necessary to place integrality requirements on some or all of the decision variables.
- The effect of integrality requirement(s) on the decision variables has very profound effect on the feasible region, and hence, on the optimal solution.
  - Simplex method cannot be used to determine the optimal solution. Hence,
  - Simplex method searches for the optimum among the corner points of the feasible region without integrality requirements.
  - A new approach to determining the optimum solution is required.
- Some of methods available for determining the optimum solution are:
  - *LP relaxation* and rounding to the nearest integer solution.
    * LP relaxation solution is at least as good as the true optimum.
  - This is intuitively very appealing but has several limitations.
    * Rounding can lead to infeasibility
    * All rounding alternatives can be infeasible!
    * The set of rounding alternatives may not include the optimal solution
    * Not practical for large scale problems. For a problem with n decision variables, 2^n alternatives need to be evaluated!
  - Cutting Plane Method
    * A modified simplex method.
    * Systematically eliminates (cuts out) parts of the feasible region which do not contain integer solution.
    * Simplex method continues after each cutting plane is added until it ends at a corner point which satisfies all the integrality requirements.
  - Branch and Bound Method
  - LINDO implements Branch and Bound Method

Formulation of an Integer Programming model is exactly the same as that for an LP model with an additional statement indicating the list of variables that need to take integer values.

- Integer Programing models can be:
  - Pure Integer Programming models
    - All variables are integer valued.
  - Mixed Integer Programming models
    - Only some of the variables are integer valued.
  - A third type uses *binary* variables.
    - *Binary* variables are defined to be either a *1* or a *0*.
    - *Binary* variables are useful in modeling a variety of *Yes-or-No* type of decisions.
    - They can be useful in modeling several situations that can not be effectively modeled using traditional LP/IP models such as:
      - Dependent Projects
      - Either-OR decisions
      - set covering problems
      - Fixed Cost Problems
      - m out of k constraints

Formulation Exercises of Integer Programming

1. A manager of a firm's research and development is currently reviewing a set of proposals prepared by her staff. She must soon decide which proposals to fund. The table below displays each proposal's net present value (NPV) and, for each of the next three years, each proposal's requirements for cash and labor:

Proposal	NPV ($000,000)	Cash Requirements ($000)			Labor Requirements (man-years)		
		Year 1	Year 2	Year 3	Year 1	Year 2	Year 3
1	5.0	200	300	300	1	2	2
2	2.8	200	200	0	2	2	0
3	4.8	300	300	0	2	2	0
4	6.0	0	300	300	0	1	2
5	5.4	100	500	400	1	3	3
Maximum Availability		700	800	900	4	6	7

   (a) Formulate an Integer Linear Program (ILP) to maximize the total NPV of the approved proposals.

   (b) What modifications to the ILP are necessary under each of the following assumptions. Here, as you consider each assumption, you should ignore the others.

      i. At most two of proposals 1, 3, and 5 can be approved.

      ii. Exactly one of proposals 1, 2, and 4 must be approved.

      iii. Proposal 2 cannot be approved unless proposal 4 is also approved.

      iv. Proposal 3 cannot be approved unless both proposals 1 and 5 are also approved.

      v. If proposals 2 and 3 are both approved, then proposal 4 must also be approved.

      vi. If proposals 1 and 2 and approved, proposal 3 cannot be approved.

2. A certain Job shop has four operators and four machines. Based on the background and training of the operators and on the nature of the machines the Industrial Engineering department estimated the cost of assigning each operator to each of the machines as given below. Determine a minimum cost assignment of the operators to the machines.

operator	Machine A	B	C	D
1	$20	$30	$32	$36
2	28	26	32	20
3	22	18	16	36
4	26	26	22	18

3. Super Sunday, the once-a-year TV extravaganza focused on the activities of the NFL Super Bowl, has almost become a tradition. CBS has received the TV contract for this year's festivities. Producers have identified 12 potential camera locations within the stadium. They have also identified 25 stadium areas that may require camera coverage during the pregame, game and postgame activities. The camera locations and the stadium areas the camera can over are given below:

Camera Location	Stadium Areas
1	1,3,4,6,7
2	8,4,7,12
3	2,5,9,11,13
4	1,2,18,19,21
5	3,6,10,12,14
6	8,14,15,16,17
7	18,21,24.25
8	2,10,16,23
9	1,6,11
10	20,22,24,25
11	2,4,6,8
12	1,6,12,17

CBS executives are concerned about costs for the production. Consequently, they set an objective of minimizing the number of camera locations used. In seeking this objective they want at least one camera to be available to cover each stadium area. Camera location 9 is the "blimp", and executives have decided that the blimp will be used because of viewer expectation and fascination with the shots from this location. Stadium areas 1 and 2 are locker room locations. The viewer interest in football personalities has led the executives to request that at least two camera locations be available to cover each of these areas. Formulate a 0-1 programming model to determine the minimum number of camera's needed for coverage.

4. The Big Bang Novelty Company makes three basic types of noise-makers: Toot, Wheet, and Honk. A Toot can be made in 30 minutes and has a feather attached to it. A Wheet requires 20 minutes, has two feathers, and is sprinkled with 0.5 ounces of sequin powder. The Honk requires 30 minutes, three feathers, and 1 ounce of sequin powder. The unit profits are $0.45 per Toot, $0.55 per Wheet, and $0.70 per Honk. The following resources are available: 4800 minutes of labor, 90 ounces of sequin powder, and 360 feathers.

   - If the company produces any Toots, it incurs a fixed cost of $15.
   - If the company produces any Wheets, it incurs a fixed cost of $25.
   - If the company produces any Honks, it incurs a fixed cost of $10.

   The company believes that the maximum numbers of Toots, Wheets and Honks it can sell are 200, 500 and 300 respectively. Formulate an ILP for determining the product mix that maximizes total profit subject to the resource constraints.

5. A manufacturing company produces three products each of which must be processed on two machines. The processing times for each product on each machine along with the maximum available time on the machines per week is given in the table below:

Machine	Product 1	Product 2	Product 3	Maximum Available
1	4 hours	6 hours	1 hour	2000 hours
2	2 hours	2 hours	3 hours	1500 hours

   There is a fixed cost associated with the production of each product. The table below shows the fixed costs along with unit contribution from the sale of a unit of each product.

Product	Fixed Costs	Unit Contribution
1	$100	$5
2	$150	$7
3	$ 75	$4

   Formulate an integer programming model to maximize the profit after accounting for the fixed costs.

6. A corporation is planning to produce a new product, and is considering six possible sites for its manufacturing plants. The plants that are built must serve four customer service centers. The accompanying table shows expected monthly demand for each customer service center, and potential monthly capacity for each possible manufacturing plant. Also shown are unit production and transportation costs for each possible plant/service center route, together with fixed costs per month for each possible plant. These cost figures are all in dollars.

	Customer service center					
	1	2	3	4	Fixed	Potential
Plant	variable cost per unit				costs	supply
1	10	12	13	17	\$3,000	700
2	11	9	10	14	\$3,500	750
3	15	12	8	10	\$2,600	650
4	18	15	13	9	\$2,100	550
5	13	16	14	12	\$3,900	800
6	18	11	8	16	\$2,800	600
Demand	800	600	700	500		

This corporation wants to select plant sites and develop a transportation policy so that total monthly costs are as small as possible, subject to the requirement that all demand must be met. Formulate the mathematical programming problem that must be solved.

7. A firm is considering three different products for the upcoming planning horizon. The pertinent information is given in the table below.

Product	Raw Material (lbs)	Direct Labor (hours)	Revenues
1	10	5	\$15
2	15	4	\$12
3	20	7	\$20

There is 500,000 lbs. of raw material available. The variable cost of direct labor is \$10 per hour. The fixed cost of the production facility is based on the amount of labor (a function of people working in the production facility and their equipment needs). The Industrial Engineering department estimates the following steps of fixed costs as a function of the direct labor requirements in the production process.

Fixed Costs (\$)	Direct labor Requirement
200,000	up to 30,000 hr.
300,000	30,000-50,000 hr.
500,000	50,000-100,000 hr.

Formulate an IP model to determine the optimal product mix.