

5. Find an irreducible polynomial $f(x)$ over the field \mathbb{Z}_3 with

$$\mathbb{Z}_3[x]/(f(x)) = \mathbb{F}_{243}.$$

Note that $243 = 3^5$.

Please explain your reasoning and solution in as much detail as possible.
Thank You.