

Let $\gcd(m, n) = 1$. We know that $Z_{mn} = Z_m \times Z_n$ as an additive group. Is there a ring isomorphism $Z_{mn} = Z_m \times Z_n$? Same question for the complex numbers \mathbb{C} with $\mathbb{C} = \mathbb{R} \times \mathbb{R}$ as an additive group.

Please explain your reasoning and solution in as much detail as possible.
Thank You.