

Please do with steps showing how you arrive at the final answer

- (a) The following is the application of the power method to estimate eigenvalues and eigenvectors.

Let X_n, Y_n denote the sales of two types of calculator at the n th year. At the beginning of year $n+1$, the corresponding units sold are given as

$$e_n = \begin{pmatrix} X_{n+1} \\ Y_{n+1} \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} X_n \\ Y_n \end{pmatrix} = A \begin{pmatrix} X_n \\ Y_n \end{pmatrix}$$

$$e_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

- (i) Find the eigenvalues and corresponding eigenvectors v_1 and v_2 of A. (4 marks)
- (ii) Express e_0, e_1, e_2 as linear combination of v_1 and v_2 . (3 marks)
- (iii) Find a general formula for e_n in terms of v_1 and v_2 . (1 mark)
- (iv) Find the limit of $\frac{Y_n}{X_n}$ as n approaches infinity. What does it imply to finding eigenvectors and corresponding eigenvalues by iteration. (2 mark)
- (b) Given position vector of two points A, B and C are $\underline{a} = 2\underline{i} - \underline{j} + \underline{k}$ and $\underline{b} = 2\underline{i} + \underline{j} + 3\underline{k}$, $\underline{c} = \underline{i} - 3\underline{j} + \underline{k}$
- (i) find the distance between A and B (1 mark)
- (ii) find the angle between $\underline{a} - \underline{b}$ and $\underline{a} + \underline{b}$. (3 marks)
- (ii) find a unit vector perpendicular to the plane containing A, B, C. (3 marks)
- (iii) find the Projection of AC along AB. (3 marks)