2.7 Exercises

1. 61 2. 161 3. 2016
4. −714 5. $-\frac{7}{2}$ 6. $\frac{18}{7}$
7. $5m^2 - 8m - 4$
8. $20k^2 + 8k + 4$
9. $5x - 1$; $x + 9$; $6x^2 - 7x - 20$; $\frac{3x+4}{2x-5}$; all domains are $(-\infty, \infty)$ except for that of $\frac{f}{g}$, which is $\left(-\infty, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$.
10. $-7x + 7$; $x + 5$; $12x^2 - 27x + 6$; $\frac{6-3x}{-4x+1}$; all domains are $(-\infty, \infty)$ except for that of $\frac{f}{g}$, which is $\left(-\infty, \frac{1}{4}\right) \cup \left(\frac{1}{4}, \infty\right)$.
11. $3x^2 - 4x + 3$; $x^2 - 2x - 3$; $2x^4 - 5x^3 + 9x^2 - 9x$; $\frac{2x^2 - 3x}{x^2 - x + 3}$; all domains are $(-\infty, \infty)$.
12. $5x^2 - x - 1$; $3x^2 + 5x - 5$; $4x^4 - 10x^3 - x^2 + 13x - 6$; $\frac{4x^2 + 2x - 3}{x^2 - 3x + 2}$; all domains are $(-\infty, \infty)$ except for that of $\frac{f}{g}$, which is $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$. 13. $\sqrt{4x-1} + \frac{1}{x}$; $\sqrt{4x-1} - \frac{1}{x}$; $\frac{\sqrt{4x-1}}{x}$; $x\sqrt{4x-1}$; all domains are $\left[\frac{1}{4}, \infty\right)$.
14. $\sqrt{5x-4} - \frac{1}{x}$; $\sqrt{5x-4} + \frac{1}{x}$; $-\frac{\sqrt{5x-4}}{x}$; $-x\sqrt{5x-4}$; all domains are $\left[\frac{4}{5}, \infty\right)$.
15. 7.7; 11.8; 19.5
16. 6.3; 8.2; 14.5
17. 1991–1996

Let $f(x) = 5x^2 - 2x$ and $g(x) = 6x + 4$. Find each of the following. See Example 1.

1. $(f + g)(3)$ **2.** $(f - g)(-5)$ **3.** $(fg)(4)$ **4.** $(fg)(-3)$

5. $\left(\frac{f}{g}\right)(-1)$ **6.** $\left(\frac{f}{g}\right)(4)$ **7.** $(f - g)(m)$ **8.** $(f + g)(2k)$

For the pair of functions defined, find $f + g$, $f - g$, fg, and $\frac{f}{g}$. Give the domain of each. See Example 2.

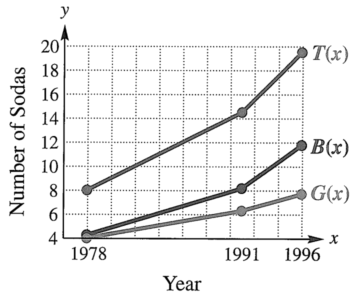
9. $f(x) = 3x + 4$, $g(x) = 2x - 5$ **10.** $f(x) = 6 - 3x$, $g(x) = -4x + 1$

11. $f(x) = 2x^2 - 3x$, $g(x) = x^2 - x + 3$

12. $f(x) = 4x^2 + 2x - 3$, $g(x) = x^2 - 3x + 2$

13. $f(x) = \sqrt{4x - 1}$, $g(x) = \frac{1}{x}$ **14.** $f(x) = \sqrt{5x - 4}$, $g(x) = -\frac{1}{x}$

Sodas Consumed by Adolescents *The graph shows the number of sodas (soft drinks) adolescents (age 12–19) drank per week from 1978–1996. $G(x)$ gives the number of sodas for girls, $B(x)$ gives the number of sodas for boys, and $T(x)$ gives the total number for both groups. Use the graph to do the following.*



Source: U.S. Department of Agriculture.

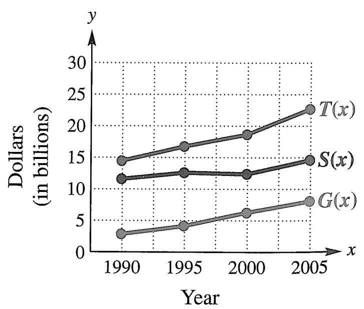
15. Estimate $G(1996)$ and $B(1996)$ and use your results to estimate $T(1996)$.

16. Estimate $G(1991)$ and $B(1991)$ and use your results to estimate $T(1991)$.

17. Use the slopes of the line segments to decide in which period (1978–1991 or 1991–1996) the number of sodas per week increased more rapidly.

18. Give a reason that might explain why girls drank fewer sodas in each of the three periods.

Science and Space/Technology Spending *The graph shows dollars (in billions) spent for general science and for space/other technologies in selected years. $G(x)$ represents the dollars spent for general science, and $S(x)$ represents the dollars spent for space and other technologies. $T(x)$ represents the total expenditures for these two categories. Use the graph to answer the following.*



Source: U.S. Office of Management and Budget.

19. Estimate $(T - S)(2000)$. What does this function represent?

20. Estimate $(T - G)(2005)$. What does this function represent?

21. In which of the categories was spending almost static for several years? In which years did this occur?

22. In which period and which category does spending for $G(x)$ or $S(x)$ increase most?

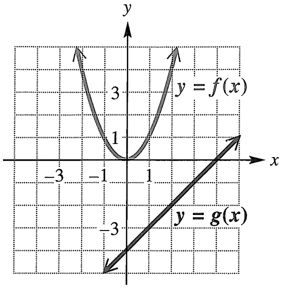
resents the dollars in
it for general science
0. about 15; It
ie dollars in billions
ace and other
s in 2005. **21.** space
chnologies;
22. space and other
s in 2000–2005
) 4 **(c)** 0 **(d)** $-\frac{1}{3}$
) −3 **(c)** 2 **(d)** −2
) −5 **(c)** 2
d
(b) −2 **(c)** 3 **(d)** −3
) 5 **(c)** 0
d
) 0 **(c)** −8 **(d)** −8

+ g) (x)	(f − g) (x)
6	−6
5	5
5	9
15	5

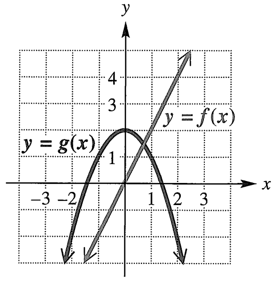
g) (x)	$\left(\frac{f}{g}\right)(x)$
0	0
0	undefined
−14	−3.5
50	2

Use the graph to evaluate each expression. See Example 3(a).

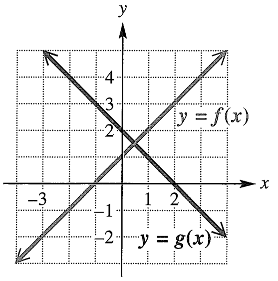
23. **(a)** $(f+g)(2)$ **(b)** $(f-g)(1)$
(c) $(fg)(0)$ **(d)** $\left(\frac{f}{g}\right)(1)$



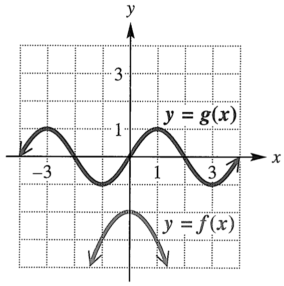
24. **(a)** $(f+g)(0)$ **(b)** $(f-g)(-1)$
(c) $(fg)(1)$ **(d)** $\left(\frac{f}{g}\right)(2)$



25. **(a)** $(f+g)(-1)$ **(b)** $(f-g)(-2)$
(c) $(fg)(0)$ **(d)** $\left(\frac{f}{g}\right)(2)$



26. **(a)** $(f+g)(1)$ **(b)** $(f-g)(0)$
(c) $(fg)(-1)$ **(d)** $\left(\frac{f}{g}\right)(1)$



In Exercises 27 and 28, use the table to evaluate each expression in parts (a)–(d), if possible. See Example 3(b).

(a) $(f+g)(2)$ **(b)** $(f-g)(4)$ **(c)** $(fg)(-2)$ **(d)** $\left(\frac{f}{g}\right)(0)$

27.

x	$f(x)$	$g(x)$
−2	0	6
0	5	0
2	7	−2
4	10	5

28.

x	$f(x)$	$g(x)$
−2	−4	2
0	8	−1
2	5	4
4	0	0

29. Use the table in Exercise 27 to complete the following table.

x	$(f+g)(x)$	$(f-g)(x)$	$(fg)(x)$	$\left(\frac{f}{g}\right)(x)$
−2				
0				
2				
4				

30.

x	$(f+g)(x)$	$(f-g)(x)$
-2	-2	-6
0	7	9
2	9	1
4	0	0

x	$(fg)(x)$	$\left(\frac{f}{g}\right)(x)$
-2	-8	-2
0	-8	-8
2	20	1.25
4	0	undefined

33. (a) $2 - x - h$ (b) $-h$ (c) -1 **34.** (a) $1 - x - h$ (b) $-h$ (c) -1
35. (a) $6x + 6h + 2$ (b) $6h$ (c) 6
36. (a) $4x + 4h + 11$ (b) $4h$ (c) 4 **37.** (a) $-2x - 2h + 5$ (b) $-2h$ (c) -2
38. (a) $1 - x^2 - 2xh - h^2$ (b) $-2xh - h^2$ (c) $-2x - h$
39. (a) $x^2 + 2xh + h^2 - 4$ (b) $2xh + h^2$ (c) $2x + h$
40. (a) $8 - 3x^2 - 6xh - 3h^2$ (b) $-6xh - 3h^2$ (c) $-6x - 3h$
41. -5 **42.** -1 **43.** 7 **44.** 0
45. 6 **46.** 10 **47.** -1
48. -2 **49.** 1 **50.** 9 **51.** 9
52. 12 **53.** 1 **54.** 12
55. $g(1) = 9$, and $f(9)$ cannot be determined from the table given.
56. 12
57. $-30x - 33$; $-30x + 52$
58. $24x + 4$; $24x + 35$
59. $4x^2 + 42x + 118$; $4x^2 + 2x + 13$
60. $-5x^2 + 20x + 18$; $-25x^2 - 10x + 6$
61. $\frac{2}{(2-x)^4}$; $(-\infty, 2) \cup (2, \infty)$; $2 - \frac{2}{x^4}$; $(-\infty, 0) \cup (0, \infty)$
62. $\frac{1}{x^2}$; $(-\infty, 0) \cup (0, \infty)$; $\frac{1}{x^2}$; $(-\infty, 0) \cup (0, \infty)$
63. $36x + 72 - 22\sqrt{x+2}$; $2\sqrt{9x^2 - 11x + 2}$
64. $\sqrt{8x^2 - 4}$ or $2\sqrt{2x^2 - 1}$; $8x + 10$

30. Use the table in Exercise 28 to complete the following table.

x	$(f+g)(x)$	$(f-g)(x)$	$(fg)(x)$	$\left(\frac{f}{g}\right)(x)$
-2				
0				
2				
4				

31. How is the difference quotient related to slope?

32. Refer to Figure 96. How is the secant line PQ related to the tangent line to a curve at point P?

For each of the functions defined as follows, find **(a)** $f(x+h)$, **(b)** $f(x+h) - f(x)$, *and* **(c)** $\frac{f(x+h) - f(x)}{h}$. *See Example 4.*

33. $f(x) = 2 - x$ **34.** $f(x) = 1 - x$ **35.** $f(x) = 6x + 2$

36. $f(x) = 4x + 11$ **37.** $f(x) = -2x + 5$ **38.** $f(x) = 1 - x^2$

39. $f(x) = x^2 - 4$ **40.** $f(x) = 8 - 3x^2$

Let $f(x) = 2x - 3$ *and* $g(x) = -x + 3$. *Find each composite function. See Example 5.*

41. $(f \circ g)(4)$ **42.** $(f \circ g)(2)$ **43.** $(f \circ g)(-2)$ **44.** $(g \circ f)(3)$

45. $(g \circ f)(0)$ **46.** $(g \circ f)(-2)$ **47.** $(f \circ f)(2)$ **48.** $(g \circ g)(-2)$

Concept Check *The tables give some selected ordered pairs for functions f and g.*

x	3	4	6
$f(x)$	1	3	9

x	2	7	1	9
$g(x)$	3	6	9	12

Find each of the following.

49. $(f \circ g)(2)$ **50.** $(f \circ g)(7)$ **51.** $(g \circ f)(3)$

52. $(g \circ f)(6)$ **53.** $(f \circ f)(4)$ **54.** $(g \circ g)(1)$

55. Why can you not determine $(f \circ g)(1)$ given the information in the tables for Exercises 49–54?

56. Extend the concept of composition of functions to evaluate $[g \circ (f \circ g)](7)$ using the tables for Exercises 49–54.

Find $(f \circ g)(x)$ *and* $(g \circ f)(x)$ *for each pair of functions. Give the domains in Exercises 61 and 62. See Examples 6 and 7.*

57. $f(x) = -6x + 9,\ g(x) = 5x + 7$ **58.** $f(x) = 8x + 12,\ g(x) = 3x - 1$

59. $f(x) = 4x^2 + 2x + 8,\ g(x) = x + 5$ **60.** $f(x) = 5x + 3,\ g(x) = -x^2 + 4x + 3$

61. $f(x) = \frac{2}{x^4},\ g(x) = 2 - x$ **62.** $f(x) = \frac{1}{x},\ g(x) = x^2$

63. $f(x) = 9x^2 - 11x,\ g(x) = 2\sqrt{x+2}$ **64.** $f(x) = \sqrt{x+2},\ g(x) = 8x^2 - 6$

f(x)	g(x)	g[f(x)]
3	2	7
1	5	2
2	7	5

x	−2	−1	0	1	2
	−1	2	0	−2	1
	0	2	1	2	0
g) (x)	0	1	−2	1	0

xercises 73–78, we give only of the many possible ways.
$g(x) = 6x - 2,\ f(x) = x^2$
$g(x) = 11x^2 + 12x,\ f(x) = x^2$
$g(x) = x^2 - 1,\ f(x) = \sqrt{x}$
$g(x) = 2x - 3,\ f(x) = x^3$
$g(x) = 6x,\ f(x) = \sqrt{x} + 12$
$g(x) = 2x + 3,$
$= \sqrt[3]{x} - 4$
$(f \circ g)(x) = 63{,}360x$
nputes the number of inches in iles.
(a) $s = \frac{x}{4}$ (b) $y = \frac{x^2}{16}$
2.25 square units
(a) $A(2x) = \sqrt{3}x^2$
$64\sqrt{3}$ square units

65. *Concept Check* Fill in the missing entries in the table.

x	f(x)	g(x)	g[f(x)]
1	3	2	7
2	1	5	
3	2		

66. *Concept Check* Suppose $f(x)$ is an odd function and $g(x)$ is an even function. Fill in the missing entries in the table.

x	−2	−1	0	1	2
f(x)			0	−2	
g(x)	0	2	1		
$(f \circ g)(x)$		1	−2		

67. Composition is an operation that is unique to functions. Is composition of functions commutative? That is, does $f \circ g = g \circ f$ for all functions f and g? Explain.

68. Describe the steps required to find the composite function $f \circ g$, given $f(x) = 2x - 5$ and $g(x) = x^2 + 3$.

For certain pairs of functions f and g, $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$. Show that this is true for each pair in Exercises 69–72.

69. $f(x) = 4x + 2,\ g(x) = \frac{1}{4}(x - 2)$

70. $f(x) = -3x,\ g(x) = -\frac{1}{3}x$

71. $f(x) = \sqrt[3]{5x + 4},\ g(x) = \frac{1}{5}x^3 - \frac{4}{5}$

72. $f(x) = \sqrt[3]{x + 1},\ g(x) = x^3 - 1$

Find functions f and g such that $(f \circ g)(x) = h(x)$. (There are many possible ways to do this.) See Example 8.

73. $h(x) = (6x - 2)^2$

74. $h(x) = (11x^2 + 12x)^2$

75. $h(x) = \sqrt{x^2 - 1}$

76. $h(x) = (2x - 3)^3$

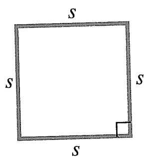
77. $h(x) = \sqrt{6x} + 12$

78. $h(x) = \sqrt[3]{2x + 3} - 4$

Solve each problem.

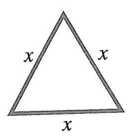
79. *Relationship of Measurement Units* The function defined by $f(x) = 12x$ computes the number of inches in x feet, and the function defined by $g(x) = 5280x$ computes the number of feet in x miles. What does $(f \circ g)(x)$ compute?

80. *Perimeter of a Square* The perimeter x of a square with side of length s is given by the formula $x = 4s$.



(a) Solve for s in terms of x.
(b) If y represents the area of this square, write y as a function of the perimeter x.
(c) Use the composite function of part (b) to find the area of a square with perimeter 6.

81. *Area of an Equilateral Triangle* The area of an equilateral triangle with sides of length x is given by the function defined by $A(x) = \frac{\sqrt{3}}{4}x^2$.



(a) Find $A(2x)$, the function representing the area of an equilateral triangle with sides of length twice the original length.
(b) Find the area of an equilateral triangle with side length 16. Use the formula $A(2x)$ found in part (a).