The University of Calgary
Faculty of Science
Department of Physics and Astronomy

Physics 321
FINAL MODULE EXAMINATION
Module 10: Waves

Friday, December 13, 2002
3:30-5:00 p.m.
Room SB 142

TIME: 90 minutes

Module summary sheet is provided with the question paper. Otherwise exam is closed book. Calculators permitted.


Do all 6 problems in Part A [6 x 6 marks each = 36 marks]

Choose ONE of the problems in section B. [9 marks]. If you attempt both problems in part B, the one with the higher mark will be counted.

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**do ALL**

| B7 | 9 |
| B8 | 9 |

**ONE only**

| TOT | 45 |

**nued....**
Part A. Do all six problems in this section.

A1. The expression given below represents a transverse harmonic wave travelling along a string which is stretched along the horizontal x-axis. The expression yields the transverse wave displacement $y$ in metres when $x$ is entered in metres and the time $t$ in seconds.

$$y(x,t) = 0.05 \sin(60t + 2x + 1)$$

For 1 mark each, answer the following questions (be sure to give physical units with your answers):

(a) What are the frequency and wavelength of this wave? [2]
(b) What is the propagation speed $c$ of this wave? [1]
(c) In what direction is this wave travelling? [1]
(d) What is the maximum transverse velocity achieved by any particle of the string? [2]
A2. On a winter day at a location near Calgary, a layer of cold Arctic air at \(-40^\circ C\) occupies a valley bottom. Above the cold air is a layer of warmer Pacific air at \(+20^\circ C\). To a good approximation, the air pressure in both layers can be assumed to be the same and equal to \(8.00 \times 10^4\) Pa.

\[\begin{array}{c}
\text{+20° C} \\
\text{−40° C}
\end{array}\]

[3] (a) Find the numerical value of the ratio of acoustic impedances of the two layers. Which has the higher impedance? (Recall gas density is inversely proportional to temperature at constant pressure.)

[1] (b) A sound wave is beamed vertically upward towards the horizontal interface between the cold air and the warmer air. Is the reflection coefficient at the interface positive or negative? Explain very briefly.

[2] (c) What fraction of the energy carried by the sound wave of part (b) is reflected from the interface?
A3. An isotropic point source of sound waves produces an intensity level of 83.0 dB at a distance of 1.00 m from the source. The source is surrounded by a uniform medium consisting of air at 20°C with sound speed 344 m/s.

(a) What is the intensity level (in dB) 1000 m from the source, assuming absorption of 0.01 dB/m in the surrounding air?

(b) What is the acoustic intensity (in W/m²) at a distance of 1000 m from the source?

(c) Calculate the total power being emitted by the source at any instant.

($I_{ref}$ for the decibel scale is $1 \times 10^{-12}$ W/m².)
A4. The diagram shows a plot of a triangular transverse wave pulse as a function of $x$ at $t=0$. Note that the vertical scale on the plot is different from the horizontal scale. The pulse is moving to the right on a string under tension 600 N at wave speed $c=40 \text{ m/s}$.

Using the axes provided, plot the transverse particle velocity $v_y$ and the transverse force $F_y$ for this pulse as functions of $x$ at $t=0$. Be careful to label your vertical axes with appropriate scales. Scales should include physical units.
A5. A liquid has bulk modulus $9.1 \times 10^8$ Pa and density $0.81 \times 10^3$ kg/m$^3$. A plane harmonic acoustic travelling wave of frequency $1000$ Hz creates an acoustic intensity of $1.00 \times 10^{-6}$ W/m$^2$ at a point $P$ in the liquid.

[1.5] (a) What is the speed of acoustic waves in this liquid?
[1.5] (b) What is the acoustic impedance of this liquid?
[1.5] (c) At an instant of time when the velocity of the fluid element at $P$ is zero, what are

[1.5] (i) the pressure fluctuation at $P$?
[1.5] (ii) the absolute value of the displacement of the fluid element at $P$?
A6. A guitar string has mass per unit length 3.00 grams/metre and is stretched under a tension of 1500N with its ends fixed a distance 80.0 cm apart.

(a) What is the frequency of the fundamental mode of vibration (i.e., the 1st harmonic) of the string under these conditions?

(b) Suppose the x-axis below lies along the guitar string, with the fixed ends at \( x=0 \) and \( x=80 \text{ cm} \). Sketch the shape of the displacement \( y \) of the string when it is vibrating in its second overtone (i.e., 3rd harmonic) under the given tension. Assume the particle at \( x=20 \text{ cm} \) has a negative displacement, as indicated.

(c) Referring to the situation you drew in part (b), suppose that the displacement of the particle at \( x=20 \text{ cm} \) is \(-0.800 \text{ mm}\) at the instant depicted in the figure. What then is the maximum displacement to be found anywhere along the guitar string at this same instant?
**Part B. [9 marks] Attempt ONE of the two problems below.** Use the following two blank pages for your answer.

**B7.** The transverse wave pulse shown below travels on a stretched string in the positive \( x \)-direction at wave propagation speed \( 1 \text{ m/s} \). The tension in the string is \( 1 \text{ N} \). The displacement \( y \) of the pulse at \( t=0 \) is given by the function to the right of the figure. This function returns the value of \( y \) in \textit{metres} when \( x \) is entered in \textit{metres}.

\[
y(x,0) = \begin{cases} 
0, & x < -1 \\
x^2, & -1 < x \leq 0 \\
x^2/2, & 0 < x \leq \sqrt{2} \\
0, & x > +\sqrt{2}
\end{cases}
\]

\( y = 1 \text{ m} \) 

(a) Find the particle velocity at \( x=\sqrt{2} \text{ m} \) as a function of time, i.e., find \( v_y(\sqrt{2},t) \). Draw a reasonably accurate plot of \( v_y(\sqrt{2},t) \) versus \( t \).

(b) Find the total energy (in joules) being transported by this wave pulse.

**OR**

**B8.** Two loudspeakers, \( A \) and \( B \), are positioned on the \( x \)-axis at \( x = 0 \text{ m} \) and \( x = +1.5 \text{ m} \). The loudspeakers act as \textit{isotropic point sources}, operating in \textit{phase} and emitting pure tones of frequency \( 340 \text{ Hz} \). Each loudspeaker emits a power of \( 10.0 \text{ W} \) uniformly in all directions. The speed of sound in air is \( 340 \text{ m/s} \), and absorption in the air may be neglected.

(a) Find the resultant acoustic intensity produced by the loudspeakers at point \( P \) located on the \( x \)-axis at \( x = +20.0 \text{ m} \). Note that \textit{interference effects} must be considered in calculating this intensity, and that \( 1/r^2 \) attenuation (i.e. geometrical spreading of the sound waves) should be taken into account. Assume that there is no blocking of the sound transmission from the more distant speaker (\( A \)) by the one nearer to \( P \).

(b) If you started at point \( P \) and travelled along the circumference of the circle shown in the diagram, what distance (arc length) would you have to travel before you encountered a local maximum in the acoustic intensity due to the two coherent sources?
End of test

RBH 2002-12-05

continued.....