

1. An electric field intensity,  $\vec{E}$ , in free space ( $\mu_0 = \mu$ ,  $\epsilon = \epsilon_0$ ) is given by

$$\vec{E} = 100 \cos(10^8 t + \beta x) \hat{a}_y \text{ [V/m]}$$

1.1. Find the direction of wave propagation?

1.2. Calculate the wave number  $\beta$ , and the time it takes to travel a distance of  $2\lambda$ ?

1.3. Sketch the wave at  $t=0, \frac{T}{4}, \frac{T}{2}, \frac{3T}{4}, 2T$ .

Hint.  $c = 3 \times 10^8 \text{ [m/s]}$ ,  $\lambda = \frac{2\pi}{\beta}$ ,  $\beta = \frac{\omega}{v} \text{ [rad/m]}$

2. In a free space, the magnetic field intensity,  $\vec{H}$  is given by:

$$\vec{H} = 10 \cos(\alpha \times 10^8 t - k x) \hat{a}_y \text{ [A/m]}$$

2.1. calculate the phase constant  $k$ , the wavelength  $\lambda$ , and the period  $T$ ?

2.2. Find the time  $t_1$ , it takes this wave to travel a distance  $\frac{\lambda}{8}$ ?

2.3. Sketch the wave at time  $t_1$ .

Hint.  $T = \frac{1}{f}$ ,  $\lambda = v \cdot T$ ,  $\beta = k = \frac{2\pi}{\lambda} = \frac{\omega}{v} \text{ [rad/m]}$ .

3. In a material for which  $\epsilon = 2\epsilon_0$ ,  $\mu = \mu_0$ , and  $\sigma = 0$ , the magnetic field intensity  $\vec{H}$  is given by:

$$\vec{H} = 2 \cos(10^6 t + \beta x) \hat{a}_z$$

3.1. Determine the displacement current,  $\vec{J}_d$

3.2. Find the displacement vector  $\vec{D}$ , and the wave number  $\beta$ .

Hint.  $\vec{J} = 0$ ,  $\nabla \times \vec{H} = \vec{J}_d$ ,  $\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$  so  $\vec{D} = \int \vec{J}_d dt$ ,  $\beta = \omega \sqrt{\mu \epsilon}$ .

Extra credit:

4. For the following time-varying EM field

$$\vec{E} = 30 \sin \omega x \cdot \sin(kz - \omega t) \hat{a}_y \text{ [V/m]}, k^2 = \omega^2 \epsilon_0 \mu_0 - 4$$

4.1. Show that  $\vec{E}$  satisfies Maxwell's equations in free space.

4.2. Find the corresponding  $\vec{H}$  and  $\vec{J}_d$ .

Hint.  $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ ,  $\nabla \cdot \vec{H} = \vec{J}_d$ ,  $\nabla \cdot \vec{D} = \rho_v$ ,  $\nabla \cdot \vec{B} = 0$ ,  $\rho_0 = 0$

$$\vec{H} = -\frac{1}{\mu_0} \int_t \nabla \times \vec{E} dt, \quad \vec{J}_d = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

1.1 the wave propagates in  $-\hat{a}_x$  direction

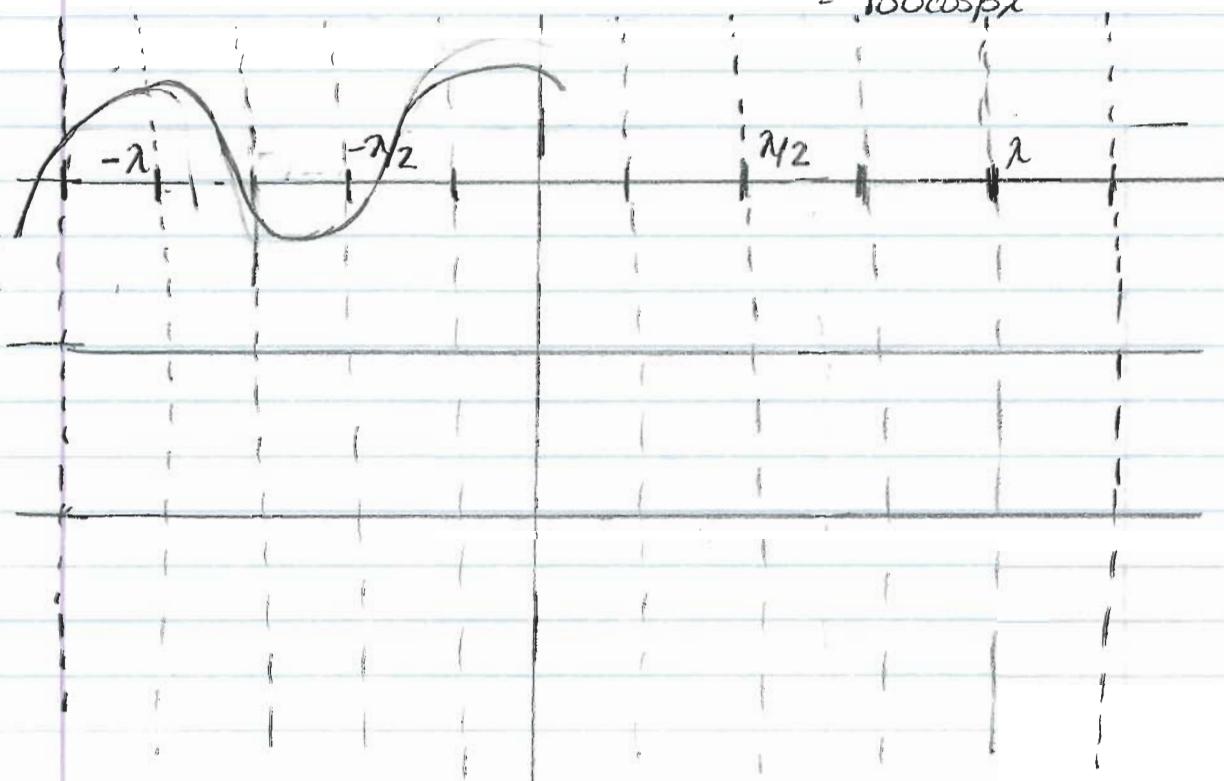
1.2  $\beta = \frac{\omega}{v} = \frac{\omega}{c} = \frac{10 \times 10^8}{3 \times 10^8} = \underline{0.333 \text{ rad/m}}$  Ans.

XRD

1.3 @  $t=0$   $E_y = 100 \cos \beta x$

@  $t=\pi/4$   $E_y = 100 \cos (\omega \frac{2\pi}{4w} + \beta x) = 100 \cos (\beta x + \pi/2)$

@  $t=\pi/2$   $E_y = 100 \cos (\omega \frac{2\pi}{2w} + \beta x) = 100 \cos (\beta x + \pi) = -100 \cos \beta x$



I need help completing that sketch.

Also if my answers are not correct, tell me what the right answer is and how to get it. Thanks.