

Problem Set 2
Random Variables

1. A die is loaded so that the probability of any side showing is proportional to the number on that side. If the die is rolled and you win 1 for every dot showing, what is the probability distribution for X , the number of dollars won? What is the probability X is less than 4?
2. Parts that are connected in a system do not fail independently of one another. In particular, two interconnected parts, A and B, fail in the following pattern. On any given day there is a 20% chance that B will fail, while if A fails, there is a .7 probability that B will fail on the same day. The probability that both parts fail on the same day is .07. On any given day, what is the most likely number of parts to fail?
3. In a particular FMS (flexible manufacturing system) setup, it can take either 1, 2, 3, or 4 hours to change over a production line. If a changeover takes at least 3 hours, there is a 90% probability that it will take 4 hours. If a changeover takes at least 2 hours, there is a .5 probability that it will take at least 3 hours. The probability that a changeover is completed in 1 hour is .20. Let X be the number of hours required for a changeover. What is the probability distribution for X ?
4. A continuous distribution with a constant density is called uniform (see Section 5.5. of MBB textbook). For instance, suppose that the daily high temperature in Durham in July is uniformly distributed between 87.5 degrees and 102.5 degrees. Then the probability density function for the daily high temperature X is given by $f(x) = 1/15$; $87.5 \leq x \leq 102.5$, and $f(x) = 0$ elsewhere. Draw this probability density function and verify that the area under the curve defined by the density function equals one. What is the probability that the daily high temperature on July 7 is greater than 100? What is the probability that the daily high temperature on July 7 is greater than or equal to 100?
5. An oil wildcatter owns drilling rights at two widely separated locations. After consulting a geologist, he feels that at each location the odds against discovering oil if a well is drilled are 9-to-1. A well costs 100,000 to drill, and this is a total loss if no oil is found. On the other hand, if oil is discovered, rights to the oil can be sold for \$1,600,000. The wildcatter has 100,000 available for drilling expenses. Find the mean and standard deviation of the wildcatter's profit
 - (a) if the 100,000 is used to drill a single well,
 - (b) if the wildcatter finds a partner to share costs and profits equally (each will receive 1/2 of the final profit, positive or negative) and their pooled funds are used to drill wells in two different locations.(Hint: Work with profits in units of 100,000 to simplify calculations.)
6. A company manufactures wooden furniture. The pieces of wood for a piece of furniture are joined by wooden dowels rather than by any metal fasteners, and the process is not too fast. Once the necessary pieces of wood for a table are cut, two tasks remain: the table is assembled, then it is stained. Let A represent the amount of time (in working days) it takes to assemble a table, and suppose that the distribution of A can be represented by the *continuous* probability density function $f(a) = 0.5$ for $1 \leq a \leq 3$ (and 0 elsewhere). The time (in working days) required to stain a table is denoted by S , and like A , it is uncertain, because the staining department may be too busy to handle a table as soon as it is received. The distribution of S can be represented by the *discrete* probability distribution: $P(s) = (6-s)/10$ for $s = 2, 3, 4, 5$ (and 0 elsewhere). Note that the assembly time can take any weird number of days (e.g., 1.57 days) whereas staining takes exactly 2,3,4, or 5 days.
 - (a) Find the probability that the assembly time for a table is more than 2 but less than 4 working days.
 - (b) Find the probability that the staining time is less than 3.5 days.
 - (c) Find the expected total time for the table to be both assembled and stained.
 - (d) What is the probability that the total time for assembly and staining is more than 4.5 working days?