; $\dfrac{\log 4}{\log\left(\frac{1}{3}\right)}$; $\dfrac{\ln 4}{\ln\left(\frac{1}{3}\right)}$

9} **6.** {1.7925}

3} **8.** {3.6674}

4} **10.** {−13.2571}

63} **12.** {−.4849}

27} **14.** {2.1023}

. ∅ **17.** {2} **18.** {3}

0112} **20.** {−75.1209}

22. {−5} **23.** ∅

. {1} **26.** $\left\{\frac{4}{3}\right\}$

28. ∅ **29.** {25}

31. {4} **32.** {1}

34. $\left\{\frac{15}{7}\right\}$

.5314} **36.** {−2.4874}

38. {6} **39.** {4}

} **41.** {1, 100}

2} **44.** any real

less than or equal to $\frac{7}{4}$

$-\frac{2}{R}\ln\left(1-\frac{RI}{E}\right)$

$^{(p-r)/k}$ **47.** $x = e^{k/(p-a)}$

$-\frac{1}{k}\log\frac{T-T_0}{T_1-T_0}$

ne power rule for

s, $(a^m)^n = a^{mn}$.

$- 1)(e^x - 3) = 0$

3}

Solve each equation. When solutions are irrational, give them as decimals correct to four decimal places. See Examples 1–6.

5. $3^x = 6$
6. $4^x = 12$
7. $6^{1-2x} = 8$
8. $3^{2x-5} = 13$
9. $2^{x+3} = 5^x$
10. $6^{x+3} = 4^x$
11. $e^{x-1} = 4$
12. $e^{2-x} = 12$
13. $2e^{5x+2} = 8$
14. $10e^{3x-7} = 5$
15. $2^x = -3$
16. $3^x = -6$
17. $e^{8x} \cdot e^{2x} = e^{20}$
18. $e^{6x} \cdot e^x = e^{21}$
19. $100(1.02)^{x/4} = 200$
20. $500(1.05)^{x/4} = 200$
21. $\ln(6x + 1) = \ln 3$
22. $\ln(7 - x) = \ln 12$
23. $\log 4x - \log(x - 3) = \log 2$
24. $\ln(-x) + \ln 3 = \ln(2x - 15)$
25. $\log(2x - 1) + \log 10x = \log 10$
26. $\ln 5x - \ln(2x - 1) = \ln 4$
27. $\log(x + 25) = 1 + \log(2x - 7)$
28. $\log(11x + 9) = 3 + \log(x + 3)$
29. $\log x + \log(x - 21) = 2$
30. $\log x + \log(3x - 13) = 1$
31. $\ln(5 + 4x) - \ln(3 + x) = \ln 3$
32. $\ln(2x + 5) + \ln x = \ln 7$
33. $\log_6 4x - \log_6(x - 3) = \log_6 12$
34. $\log_2 3x + \log_2 3 = \log_2(2x + 15)$
35. $5^{x+2} = 2^{2x-1}$
36. $6^{x-3} = 3^{4x+1}$
37. $\ln e^x - \ln e^3 = \ln e^5$
38. $\ln e^x - 2\ln e = \ln e^4$
39. $\log_2(\log_2 x) = 1$
40. $\log x = \sqrt{\log x}$
41. $\log x^2 = (\log x)^2$
42. $\log_2 \sqrt{2x^2} = \frac{3}{2}$

43. Suppose you overhear the following statement: "I must reject any negative answer when I solve an equation involving logarithms." Is this correct? Write an explanation of why it is or is not correct.

44. What values of x could not possibly be solutions of the following equation?

$$\log_a(4x - 7) + \log_a(x^2 + 4) = 0$$

Solve each equation for the indicated variable. Use logarithms to the appropriate bases. See Example 7.

45. $I = \frac{E}{R}(1 - e^{-Rt/2})$ for t
46. $r = p - k\ln t$ for t
47. $p = a + \frac{k}{\ln x}$ for x
48. $T = T_0 + (T_1 - T_0)10^{-kt}$ for t

Relating Concepts

For individual or collaborative investigation

(Exercises 49–54)

Earlier, we introduced methods of solving quadratic equations and showed how they can be applied to equations that are not actually quadratic, but are quadratic in form. Consider the following equation and ***work Exercises 49–54 in order.***

$$e^{2x} - 4e^x + 3 = 0$$

49. The expression e^{2x} is equivalent to $(e^x)^2$. Why is this so?

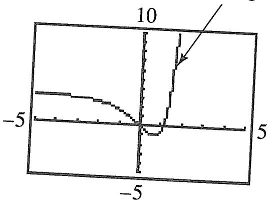
50. The given equation is equivalent to $(e^x)^2 - 4e^x + 3 = 0$. Factor the left side of this equation.

51. Solve the equation in Exercise 50 by using the zero-factor property. Give exact solutions.

(continued)

52. The graph intersects the x-axis at 0 and $\ln 3 \approx 1.099$.

$y = e^{2x} - 4e^{x} + 3$



53. $(-\infty, 0) \cup (\ln 3, \infty)$
54. $(0, \ln 3)$
55. $f^{-1}(x) = \ln(x + 4) - 1$; domain: $(-4, \infty)$; range: $(-\infty, \infty)$
56. $f^{-1}(x) = \frac{1}{3}e^{x/2}$; domain: $(-\infty, \infty)$; range: $(0, \infty)$
57. $(27, \infty)$ 58. $(1, 5)$
59. $\{1.52\}$ 60. $\{-.93, 1.35\}$
61. $\{0\}$ 62. $\{.69, 1.10\}$
63. $\{2.45, 5.66\}$ 64. $\{.23\}$
65. during 2011
66. (a) 11.6451 m per sec (b) 2.4823 sec

---

52. Support your solution(s) in Exercise 51 by graphing $y = e^{2x} - 4e^{x} + 3$ with a calculator.

53. Use the graph from Exercise 52 to identify the x-intervals where $y > 0$. These intervals give the solutions of $e^{2x} - 4e^{x} + 3 > 0$.

54. Use the graph from Exercise 52 and your answer to Exercise 53 to give the intervals where $e^{2x} - 4e^{x} + 3 < 0$.

Find $f^{-1}(x)$, and give the domain and range.

55. $f(x) = e^{x+1} - 4$ **56.** $f(x) = 2 \ln 3x$

Solve each inequality with a graphing calculator by rearranging terms so that one side is 0, then graphing the expression on the other side as y and observing from the graph where y is positive or negative as applicable. These inequalities are studied in calculus to determine where certain functions are increasing.

57. $\log_3 x > 3$ **58.** $\log_x .2 < -1$

Use a graphing calculator to solve each equation. Give irrational solutions correct to the nearest hundredth.

59. $e^{x} + \ln x = 5$ **60.** $e^{x} - \ln(x + 1) = 3$ **61.** $2e^{x} + 1 = 3e^{-x}$

62. $e^{x} + 6e^{-x} = 5$ **63.** $\log x = x^2 - 8x + 14$ **64.** $\ln x = -\sqrt[3]{x + 3}$

(Modeling) *Solve each application. See Example 8.*

65. ***Average Annual Public University Costs*** The table shows the cost of a year's tuition, room and board, and fees at a public university from 2000–2008. (*Note:* The amounts for 2002–2008 are projections.) Letting y represent the cost and x represent the number of years since 2000, we find that the function defined by

$$f(x) = 8160(1.06)^{x}$$

models the data quite well. According to this function, when will the cost in 2000 be doubled?

Year	Average Annual Cost
2000	\$7,990
2001	\$8,470
2002	\$9,338
2003	\$9,805
2004	\$10,295
2005	\$10,810
2006	\$11,351
2007	\$11,918
2008	\$12,514

Source: www.princetonreview.com

66. ***Race Speed*** At the World Championship races held at Rome's Olympic Stadium in 1987, American sprinter Carl Lewis ran the 100-m race in 9.86 sec. His speed in meters per second after t seconds is closely modeled by the function defined by

$$f(t) = 11.65(1 - e^{-t/1.27}).$$

(*Source:* Banks, Robert B., *Towing Icebergs, Falling Dominoes, and Other Adventures in Applied Mathematics,* Princeton University Press, 1998.)

(a) How fast was he running as he crossed the finish line?

(b) After how many seconds was he running at the rate of 10 m per sec?

70. **(a)** $R \approx 4.4 \text{ w/m}^2$
(b) $T \approx 4.5°F$. This is less than that predicted by Arrhenius in 1896; however, his values are still consistent with some current computer models.
71. 2.6 yr **72.** 5.55 yr
73. 6.48% **74.** 4.27%

(a) Use the equation $R = 6.3 \ln \frac{C}{C_0}$ to determine the radiative forcing R (in watt per square meter) expected by the IPCC if the carbon dioxide level in the atmosphere doubles from its preindustrial level.

(b) Determine the global temperature increase T predicted by the IPCC if the carbon dioxide levels were to double. (*Hint:* $T(R) = 1.03R$.)

For Exercises 71–74, refer to the formula for compound interest given in Section 4.2.

$$A = P\left(1 + \frac{r}{m}\right)^{tm}$$

71. ***Interest on an Account*** Tom Tupper wants to buy a \$30,000 car. He has saved \$27,000. Find the number of years (to the nearest tenth) it will take for his \$27,000 to grow to \$30,000 at 4% interest compounded quarterly.

72. ***Investment Time*** Find t to the nearest hundredth if \$1786 becomes \$2063 at 2.6%, with interest compounded monthly.

73. ***Interest Rate*** Find the interest rate that will produce \$2500 if \$2000 is left at interest compounded semiannually for 3.5 yr.

74. ***Interest Rate*** At what interest rate will \$16,000 grow to \$20,000 if invested for 5.25 yr and interest is compounded quarterly?

4.6 Applications and Models of Exponential Growth and Decay

The Exponential Growth or Decay Function ▪ **Growth Function Models** ▪ **Decay Function Models**

The Exponential Growth or Decay Function In many situations that occur in ecology, biology, economics, and the social sciences, a quantity changes at a rate proportional to the amount present. In such cases the amount present at time t is a special function of t called an **exponential growth or decay function.**

Looking Ahead to Calculus

The exponential growth and decay function formulas are studied in calculus in conjunction with the topic known as *differential equations.*

Exponential Growth or Decay Function

Let y_0 be the amount or number present at time $t = 0$. Then, under certain conditions, the amount present at any time t is modeled by

$$y = y_0 e^{kt},$$

where k is a constant.

When $k > 0$, the function describes growth; in Section 4.2, we saw examples of exponential growth: compound interest and atmospheric carbon dioxide, for example. When $k < 0$, the function describes decay; one example of exponential decay is radioactivity.

Growth Function Models The amount of time it takes for a quantity that grows exponentially to become twice its initial amount is called its **doubling time.** The first two examples involve doubling time.