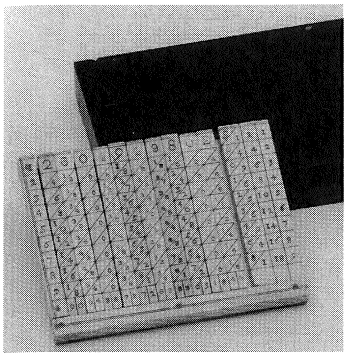
The second statement in the theorem will be useful in Sections 4.5 and 4.6 when we solve other logarithmic and exponential equations.



Napier's Rods

Source: IBM Corporate Archives

CONNECTIONS The search for making calculations easier has been a long, ongoing process. Machines built by Charles Babbage and Blaise Pascal, a system of "rods" used by John Napier, and slide rules were the forerunners of today's calculators and computers. The invention of logarithms by John Napier in the sixteenth century was a great breakthrough in the search for easier calculation methods.

Since logarithms are exponents, their properties allowed users of tables of common logarithms to multiply by adding, divide by subtracting, raise to powers by multiplying, and take roots by dividing. Although logarithms are no longer used for computations, they still play an important role in higher mathematics.

For Discussion or Writing

1. To multiply 458.3 by 294.6 using logarithms, we add $\log_{10} 458.3$ and $\log_{10} 294.6$, then find 10 raised to the sum. Perform this multiplication using the log key and the 10^x key on your calculator.* Check your answer by multiplying directly with your calculator.
2. Try division, raising to a power, and taking a root by this method.

1. $\log_{10} 458.3 \approx 2.661149857$
$+ \log_{10} 294.6 \approx 2.469232743$
≈ 5.130382600
$10^{5.130382600} \approx 135{,}015.18$
A calculator gives
$(458.3)(294.6) = 135{,}015.18.$
2. Answers will vary.

4.3 Exercises

1. (a) C (b) A (c) E (d) B (e) F (f) D **2.** (a) F (b) B (c) A (d) D (e) C (f) E
3. $\log_3 81 = 4$ **4.** $\log_2 32 = 5$
5. $\log_{2/3} \frac{27}{8} = -3$
6. $\log_{10} .0001 = -4$
7. $6^2 = 36$ **8.** $5^1 = 5$
9. $(\sqrt{3})^8 = 81$ **10.** $4^{-3} = \frac{1}{64}$

Concept Check In Exercises 1 and 2, match the logarithm in Column I with its value in Column II. Remember that $\log_a x$ *is the exponent to which a must be raised in order to obtain x.*

1.

I	II
(a) $\log_2 16$	A. 0
(b) $\log_3 1$	B. $\frac{1}{2}$
(c) $\log_{10} .1$	C. 4
(d) $\log_2 \sqrt{2}$	D. -3
(e) $\log_e \frac{1}{e^2}$	E. -1
(f) $\log_{1/2} 8$	F. -2

2.

I	II
(a) $\log_3 81$	A. -2
(b) $\log_3 \frac{1}{3}$	B. -1
(c) $\log_{10} .01$	C. 0
(d) $\log_6 \sqrt{6}$	D. $\frac{1}{2}$
(e) $\log_e 1$	E. $\frac{9}{2}$
(f) $\log_3 27^{3/2}$	F. 4

For each statement, write an equivalent statement in logarithmic form.

3. $3^4 = 81$ **4.** $2^5 = 32$ **5.** $\left(\frac{2}{3}\right)^{-3} = \frac{27}{8}$ **6.** $10^{-4} = .0001$

For each statement, write an equivalent statement in exponential form.

7. $\log_6 36 = 2$ **8.** $\log_5 5 = 1$ **9.** $\log_{\sqrt{3}} 81 = 8$ **10.** $\log_4 \frac{1}{64} = -3$

*In this text, the notation $\log x$ is used to mean $\log_{10} x$. This is also the meaning of the log key on calculators.