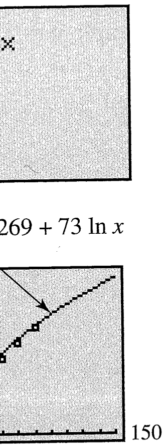

re 37

In Example 6(b), we found that $\log_2 .1 \approx -3.32$. Now we find $\log_2 .9$.

$$\log_2 .9 = \frac{\log .9}{\log 2} \approx \frac{-.0458}{.3010} \approx -.152$$

Therefore,

$$H = -[.9 \log_2 .9 + .1 \log_2 .1]$$
$$\approx -[.9(-.152) + .1(-3.32)] \approx .469.$$

Verify that $H \approx .971$ if there are 60 of one species and 40 of the other. As the proportions of n species get closer to $\frac{1}{n}$ each, the measure of diversity increases to a maximum of $\log_2 n$.

Now try Exercise 69.

At the end of Section 4.2, we saw that graphing calculators are capable of fitting exponential curves to data that suggest such behavior. The same is true for logarithmic curves. Figure 37 shows how a calculator gives the best-fitting natural logarithmic curve for the data in Exercise 65 in this section: $y = -269 + 73 \ln x$. ■

cises

2. increasing
$_5 x$ 4. $\log_4 11$
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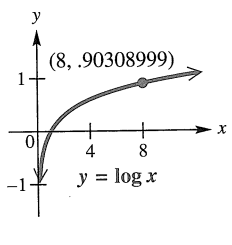
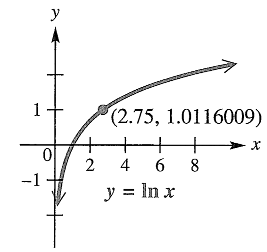
Concept Check *Answer each of the following.*

1. For the exponential function defined by $f(x) = a^x$, where $a > 1$, is the function increasing or is it decreasing over its entire domain?
2. For the logarithmic function defined by $g(x) = \log_a x$, where $a > 1$, is the function increasing or is it decreasing over its entire domain?
3. If $f(x) = 5^x$, what is the rule for $f^{-1}(x)$?
4. What is the name given to the exponent to which 4 must be raised in order to obtain 11?
5. A base e logarithm is called a(n) ________ logarithm; a base 10 logarithm is called a(n) ________ logarithm.
6. How is $\log_3 12$ written in terms of natural logarithms?
7. Why is $\log_2 0$ undefined?
8. Between what two consecutive integers must $\log_2 12$ lie?
9. The graph of $y = \log x$ shows a point on the graph. Write the logarithmic equation associated with that point.



10. ln 2.75 = 1.0116009
11. 1.5563 12. 1.8573
13. −1.3768 14. −.4962
15. 4.3010 16. −3.6990
17. 3.5835 18. 4.2767
19. −3.1701 20. −1.1426
21. 4.6931 22. −3.3069
23. 3.2 24. 8.4 25. 1.8
26. 13.5
27. 2.0×10^{-3} 28. 4.0×10^{-4}
29. 1.6×10^{-5} 30. 3.2×10^{-7}
31. poor fen 32. bog 33. rich fen 34. (a) 2.60031933 (b) 1.60031933 (c) .6003193298 (d) The whole number parts will vary, but the decimal parts will be the same.
35. 2.3219 36. 3.1699
37. −.2537 38. −.1647
39. 1.9376 40. 1.0932
41. −1.4125 42. −22.0488
43. D 44. D 45. $4v + \frac{1}{2}u$
46. $3u - 2v$ 47. $\frac{3}{2}u - \frac{5}{2}v$
48. $\frac{1}{3}u + 4v$

10. The graph of $y = \ln x$ shows a point on the graph. Write the logarithmic equation associated with that point.



Use a calculator with logarithm keys to find an approximation to four decimal places for each expression.

11. log 36	12. log 72	13. log .042	14. log .319
15. $\log(2 \times 10^4)$	16. $\log(2 \times 10^{-4})$	17. ln 36	18. ln 72
19. ln .042	20. ln .319	21. $\ln(2 \times e^4)$	22. $\ln(2 \times e^{-4})$

For each substance, find the pH from the given hydronium ion concentration. See Example 1(a).

23. grapefruit, 6.3×10^{-4}
24. crackers, 3.9×10^{-9}
25. limes, 1.6×10^{-2}
26. sodium hydroxide (lye), 3.2×10^{-14}

Find the $[H_3O^+]$ *for each substance with the given pH. See Example 1(b).*

27. soda pop, 2.7
28. wine, 3.4
29. beer, 4.8
30. drinking water, 6.5

In Exercises 31–33, suppose that water from a wetland area is sampled and found to have the given hydronium ion concentration. Determine whether the wetland is a rich fen, poor fen, or bog. See Example 2.

31. 2.49×10^{-5} 32. 2.49×10^{-2} 33. 2.49×10^{-7}

34. Use your calculator to find an approximation for each logarithm.
(a) log 398.4 (b) log 39.84 (c) log 3.984
(d) From your answers to parts (a)–(c), make a conjecture concerning the decimal values in the approximations of common logarithms of numbers greater than 1 that have the same digits.

Use the change-of-base theorem to find an approximation for four decimal places for each logarithm. See Example 6.

35. $\log_2 5$	36. $\log_2 9$	37. $\log_8 .59$	38. $\log_8 .71$
39. $\log_{\sqrt{13}} 12$	40. $\log_{\sqrt{19}} 5$	41. $\log_{.32} 5$	42. $\log_{.91} 8$

43. *Concept Check* Which of the following is the same as $2\ln(3x)$ for $x > 0$?
A. $\ln 9 + \ln x$ B. $\ln(6x)$ C. $\ln 6 + \ln x$ D. $\ln(9x^2)$

44. *Concept Check* Which of the following is the same as $\ln(4x) - \ln(2x)$ for $x > 0$?
A. $2 \ln x$ B. $\ln(2x)$ C. $\frac{\ln(4x)}{\ln(2x)}$ D. $\ln 2$

Let $u = \ln a$ *and* $v = \ln b$. *Write each expression in terms of u and v without using the* ln *function.*

45. $\ln(b^4\sqrt{a})$ 46. $\ln \frac{a^3}{b^2}$ 47. $\ln \sqrt{\frac{a^3}{b^5}}$ 48. $\ln(\sqrt[3]{a} \cdot b^4)$