

**Now try Exercise 97.**

(c) The best-fitting quartic function is defined by

$$y = -1.619x^4 + 36.09x^3 - 155.5x^2 + 218.1x + 127.$$

See Figure 36.

(d) Find the correlation coefficient values  $R^2$  for the quadratic function. The others are .9982771996 for the cubic function and 1 for the quartic function. Therefore, the quartic function provides the best fit.

**Figure 36**

Sketch the graph of each polynomial function. See Examples 1 and 2.

9.  $f(x) = 2x^4$       10.  $f(x) = \frac{1}{4}x^6$       11.  $f(x) = -\frac{2}{3}x^5$   
 12.  $f(x) = -\frac{5}{4}x^5$       13.  $f(x) = \frac{1}{2}x^3 + 1$       14.  $f(x) = -x^4 + 2$   
 15.  $f(x) = -(x+1)^3$       16.  $f(x) = (x+2)^3 - 1$       17.  $f(x) = (x-1)^4 + 2$   
 18.  $f(x) = \frac{1}{3}(x+3)^4$       19.  $f(x) = \frac{1}{2}(x-2)^2 + 4$       20.  $f(x) = \frac{1}{3}(x+1)^3 - 3$

## Exercises

2. B 3. one 4. C  
and D 6. B

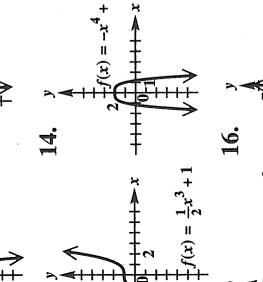
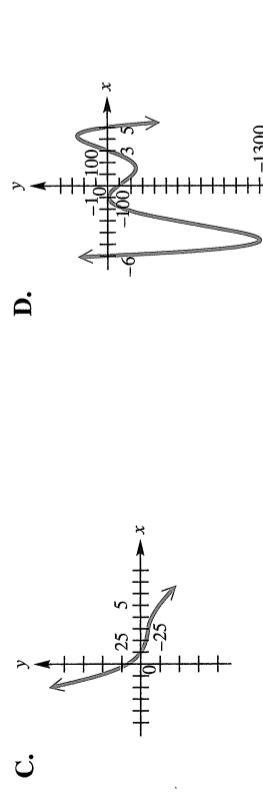
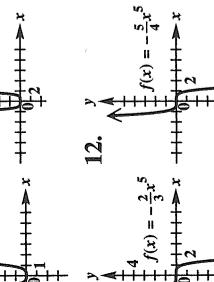
**Concept Check** Comprehensive graphs of four polynomial functions are shown in A–D. They represent the graphs of functions defined by these four equations, but not necessarily in the order listed.

$$y = x^3 - 3x^2 - 6x + 8$$

$$y = -x^3 + 9x^2 - 27x + 17$$

$$y = x^4 + 7x^3 - 5x^2 - 75x$$

Apply the concepts of this section to answer each question.



1. Which one of the graphs is that of  $y = x^3 - 3x^2 - 6x + 8$ ?  
 2. Which one of the graphs is that of  $y = x^4 + 7x^3 - 5x^2 - 75x$ ?  
 3. How many real zeros does the graph in C have?  
 4. Which one of C and D is the graph of  $y = -x^3 + 9x^2 - 27x + 17$ ? (Hint: Look at the y-intercept.)

5. Which of the graphs cannot be that of a cubic polynomial function?  
 6. Which one of the graphs is that of a function whose range is not  $(-\infty, \infty)$ ?  
 7. The function defined by  $f(x) = x^4 + 7x^3 - 5x^2 - 75x$  has the graph shown in B.  
 Use the graph to factor the polynomial.

8. The function defined by  $f(x) = -x^5 + 36x^3 - 22x^2 - 147x - 90$  has the graph shown in D. Use the graph to factor the polynomial.

Sketch the graph of each polynomial function. See Examples 1 and 2.

9.  $f(x) = 2x^4$       10.  $f(x) = \frac{1}{4}x^6$       11.  $f(x) = -\frac{2}{3}x^5$   
 12.  $f(x) = -\frac{5}{4}x^5$       13.  $f(x) = \frac{1}{2}x^3 + 1$       14.  $f(x) = -x^4 + 2$   
 15.  $f(x) = -(x+1)^3$       16.  $f(x) = (x+2)^3 - 1$       17.  $f(x) = (x-1)^4 + 2$   
 18.  $f(x) = \frac{1}{3}(x+3)^4$       19.  $f(x) = \frac{1}{2}(x-2)^2 + 4$       20.  $f(x) = \frac{1}{3}(x+1)^3 - 3$

Use an end behavior diagram  $\uparrow \downarrow \nearrow \searrow \nearrow \downarrow$ , to describe the end behavior of the graph of each polynomial function. See Example 3.

21.  $f(x) = 5x^3 + 2x^2 - 3x + 4$       22.  $f(x) = -6x^3 - 4x^2 + 2x - 1$   
 23.  $f(x) = -4x^5 + 3x^2 - 1$       24.  $f(x) = 8x^7 - x^5 + x - 1$   
 25.  $f(x) = 9x^4 - 3x^2 + x - 2$       26.  $f(x) = 12x^6 - x^5 + 2x - 2$   
 27.  $f(x) = 3 + 2x - 4x^2 - 5x^3$       28.  $f(x) = 8 + 2x - 5x^2 - 10x^4$

Graph each polynomial function. Factor first if the expression is not in factored form. See Example 4.

29.  $f(x) = x^3 + 5x^2 + 2x - 8$       30.  $f(x) = x^3 + 3x^2 - 13x - 15$   
 31.  $f(x) = x^2(x+1)(x+2)$       32.  $f(x) = x^2(x+1)(x-1)$   
 33.  $f(x) = x^2(x-2)(x+3)^2$       34.  $f(x) = x^2(x-5)(x+3)(x-1)$   
 35.  $f(x) = (3x-1)(x+2)^2$       36.  $f(x) = (4x+3)(x+2)^2$   
 37.  $f(x) = x^3 + 5x^2 - x - 5$       38.  $f(x) = x^3 + x^2 - 36x - 36$   
 39.  $f(x) = x^3 - x^2 - 2x$       40.  $f(x) = 3x^4 + 5x^3 - 2x^2$   
 41.  $f(x) = 2x^3(x^2 - 4)(x - 1)$       42.  $f(x) = x^2(x - 3)^3(x + 1)$

Use the intermediate value theorem for polynomials to show that each polynomial function has a real zero between the numbers given. See Example 5.

43.  $f(x) = 2x^2 - 7x + 4$ ; 2 and 3      44.  $f(x) = 3x^2 - x - 4$ ; 1 and 2  
 45.  $f(x) = 2x^3 - 5x^2 - 5x + 7$ ; 0 and 1      46.  $f(x) = 2x^3 - 9x^2 + x + 20$ ; 2 and 2.5  
 47.  $f(x) = 2x^4 - 4x^2 + 4x - 8$ ; 1 and 2      48.  $f(x) = x^4 - 4x^3 - x + 3$ ; 5 and 1  
 49.  $f(x) = x^4 + x^3 - 6x^2 - 20x - 16$ ; 3.2 and 3.3      50.  $f(x) = x^4 - 2x^3 - 2x^2 - 18x + 5$ ; 3.7 and 3.8  
 51.  $f(x) = x^4 - 4x^3 - 20x^2 + 32x + 12$ ; -1 and 0      52.  $f(x) = x^5 + 2x^4 + x^3 + 3$ ; -1.8 and -1.7

Show that the real zeros of each polynomial function satisfy the given conditions. See Example 6.

53.  $f(x) = x^4 - x^3 + 3x^2 - 8x + 8$ ; no real zero greater than 2  
 54.  $f(x) = 2x^5 - x^4 + 2x^3 - 2x^2 + 4x - 4$ ; no real zero greater than 1  
 55.  $f(x) = x^4 + x^3 - x^2 + 3$ ; no real zero less than -2