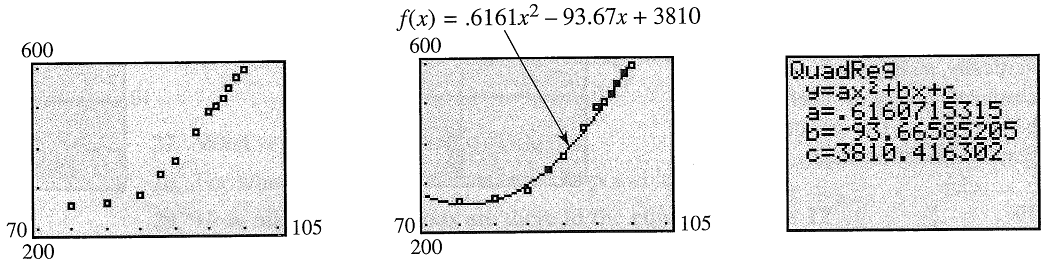
Solution

(a) The scatter diagram in Figure 15(a) suggests that a quadratic function with a positive value of a (so the graph opens up) would be a reasonable model for the data. Using quadratic regression, the quadratic function defined by $f(x) = .6161x^2 - 93.67x + 3810$ approximates the data well. See Figure 15(b). Figure 15(c) displays the quadratic regression values of a, b, and c.



(a) (b) (c)

Figure 15

(b) Since 2005 corresponds to $x = 105$, the model predicts that in 2005 the number of visits will be

$$f(105) = .6161(105)^2 - 93.67(105) + 3810 \approx 767 \text{ million.}$$

Now try Exercise 61.

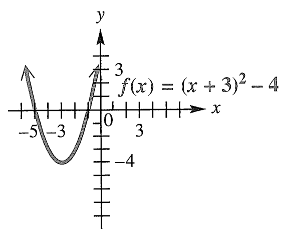
Exercises

domain: $(-\infty, \infty)$;
$[-4, \infty)$ **(b)** $(-3, -4)$
$= -3$ **(d)** 5 **(e)** $-5, -1$
domain: $(-\infty, \infty)$;
$[-4, \infty)$ **(b)** $(5, -4)$
$= 5$ **(d)** 21 **(e)** 3, 7
domain: $(-\infty, \infty)$;
$(-\infty, 2]$ **(b)** $(-3, 2)$
$= -3$ **(d)** -16 **(e)** $-4, -2$
domain: $(-\infty, \infty)$;
$(-\infty, 1]$ **(b)** $(2, 1)$
$= 2$ **(d)** -11
$\frac{- \sqrt{3}}{3}, \frac{6 + \sqrt{3}}{3}$

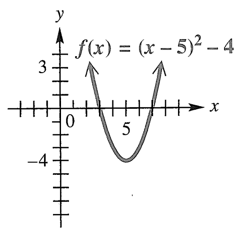
In Exercises 1–4, you are given an equation and the graph of a quadratic function. Do each of the following. See Examples 1(c) and 2–4.

(a) Give the domain and range.
(b) Give the coordinates of the vertex.
(c) Give the equation of the axis.
(d) Find the y-intercept.
(e) Find the x-intercepts.

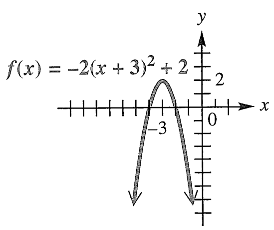
1. $f(x) = (x + 3)^2 - 4$



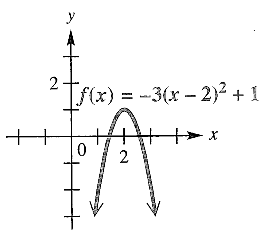
2. $f(x) = (x - 5)^2 - 4$



3. $f(x) = -2(x + 3)^2 + 2$

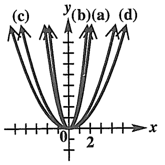


4. $f(x) = -3(x - 2)^2 + 1$



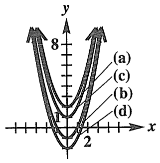
5. B **6.** A **7.** D **8.** C

9.



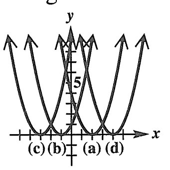
(e) If the absolute value of the coefficient is greater than 1, it causes the graph to be stretched vertically, so it is narrower. If the absolute value of the coefficient is between 0 and 1, it causes the graph to shrink vertically, so it is broader.

10.



(e) The graph of $x^2 + k$ is translated k units up if k is positive and $|k|$ units down is k is negative.

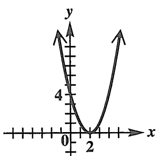
11.



(e) The graph of $(x - h)^2$ is translated h units to the right if h is positive and $|h|$ units to the left if h is negative.

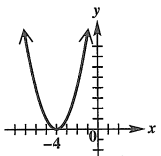
12. **(a)** D **(b)** B **(c)** C **(d)** A

13. vertex: $(2, 0)$; axis: $x = 2$; domain: $(-\infty, \infty)$; range: $[0, \infty)$



$f(x) = (x - 2)^2$

14. vertex: $(-4, 0)$; axis: $x = -4$; domain: $(-\infty, \infty)$; range: $[0, \infty)$



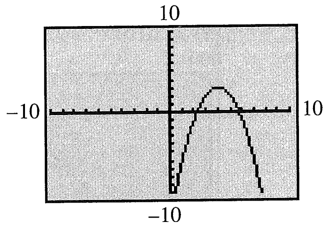
$f(x) = (x + 4)^2$

Concept Check Calculator graphs of the functions in Exercises 5–8 are shown in Figures A–D. Match each function with its graph without actually entering it into your calculator. Then, after you have completed the exercises, check your answers with your calculator. Use the standard viewing window.

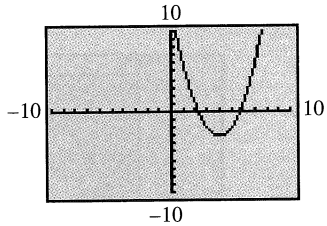
5. $f(x) = (x - 4)^2 - 3$ **6.** $f(x) = -(x - 4)^2 + 3$

7. $f(x) = (x + 4)^2 - 3$ **8.** $f(x) = -(x + 4)^2 + 3$

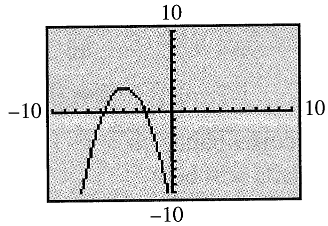
A.



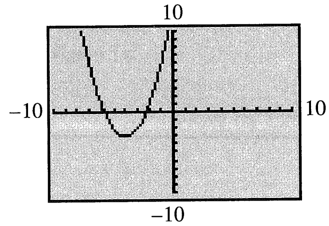
B.



C.



D.



9. Graph the following on the same coordinate system.

(a) $y = 2x^2$ **(b)** $y = 3x^2$ **(c)** $y = \frac{1}{2}x^2$ **(d)** $y = \frac{1}{3}x^2$

(e) How does the coefficient of x^2 affect the shape of the graph?

10. Graph the following on the same coordinate system.

(a) $y = x^2 + 2$ **(b)** $y = x^2 - 1$ **(c)** $y = x^2 + 1$ **(d)** $y = x^2 - 2$

(e) How do these graphs differ from the graph of $y = x^2$?

11. Graph the following on the same coordinate system.

(a) $y = (x - 2)^2$ **(b)** $y = (x + 1)^2$ **(c)** $y = (x + 3)^2$ **(d)** $y = (x - 4)^2$

(e) How do these graphs differ from the graph of $y = x^2$?

12. *Concept Check* Match each equation with the description of the parabola that is its graph.

(a) $y = (x - 4)^2 - 2$	**A.** vertex $(2, -4)$, opens down
(b) $y = (x - 2)^2 - 4$	**B.** vertex $(2, -4)$, opens up
(c) $y = -(x - 4)^2 - 2$	**C.** vertex $(4, -2)$, opens down
(d) $y = -(x - 2)^2 - 4$	**D.** vertex $(4, -2)$, opens up

Graph each quadratic function. Give the vertex, axis, domain, and range. See Examples 1–4.

13. $f(x) = (x - 2)^2$ **14.** $f(x) = (x + 4)^2$

15. $f(x) = (x + 3)^2 - 4$ **16.** $f(x) = (x - 5)^2 - 4$

17. $f(x) = -\frac{1}{2}(x + 1)^2 - 3$ **18.** $f(x) = -3(x - 2)^2 + 1$

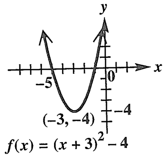
19. $f(x) = x^2 - 2x + 3$ **20.** $f(x) = x^2 + 6x + 5$

21. $f(x) = x^2 - 10x + 21$ **22.** $f(x) = 2x^2 - 4x + 5$

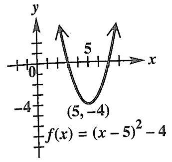
23. $f(x) = -2x^2 - 12x - 16$ **24.** $f(x) = -3x^2 + 24x - 46$

25. $f(x) = -x^2 - 6x - 5$ **26.** $f(x) = \frac{2}{3}x^2 - \frac{8}{3}x + \frac{5}{3}$

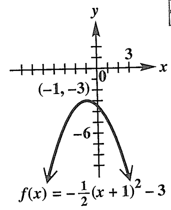
15. vertex: $(-3, -4)$; axis: $x = -3$; domain: $(-\infty, \infty)$; range: $[-4, \infty)$



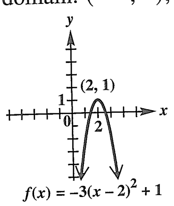
16. vertex: $(5, -4)$; axis: $x = 5$; domain: $(-\infty, \infty)$; range: $[-4, \infty)$



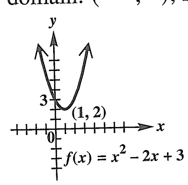
17. vertex: $(-1, -3)$; axis: $x = -1$; domain: $(-\infty, \infty)$; range: $(-\infty, -3]$



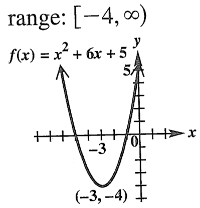
18. vertex: $(2, 1)$; axis: $x = 2$; domain: $(-\infty, \infty)$; range: $(-\infty, 1]$



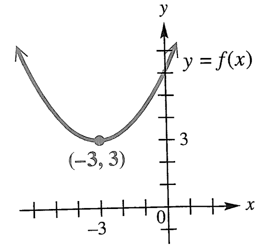
19. vertex: $(1, 2)$; axis: $x = 1$; domain: $(-\infty, \infty)$; range: $[2, \infty)$



20. vertex: $(-3, -4)$; axis: $x = -3$; domain: $(-\infty, \infty)$; range: $[-4, \infty)$



Concept Check *The figure shows the graph of a quadratic function $y = f(x)$. Use it to work Exercises 27–30.*



27. What is the minimum value of $f(x)$?

28. For what value of x is $f(x)$ as small as possible?

29. How many real solutions are there to the equation $f(x) = 1$?

30. How many real solutions are there to the equation $f(x) = 4$?

31. In Chapter 2, we saw how certain changes to an equation cause the graph of the equation to be stretched, shrunken, reflected across an axis, or translated vertically or horizontally. The order in which these changes are done affects the final graph. For example, stretching and then shifting vertically produces a graph that differs from the one produced by shifting vertically, then stretching. To see this, use a graphing calculator to graph

$$y = 3x^2 - 2 \qquad \text{and} \qquad y = 3(x^2 - 2),$$

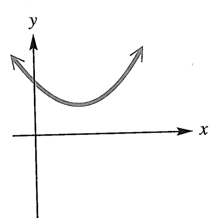
and then compare the results. Are the two expressions equivalent algebraically?

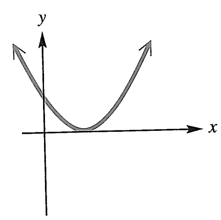
32. *Concept Check* Suppose that a quadratic function with $a > 0$ is written in the form $f(x) = a(x - h)^2 + k$. Match each statement in Column I with one of the choices A, B, or C in Column II.

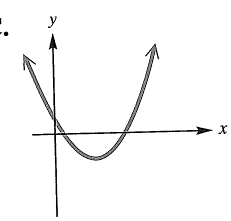
I	II
(a) k is positive.	**A.** The graph of $f(x)$ intersects the x-axis at only one point.
(b) k is negative.	**B.** The graph of $f(x)$ does not intersect the x-axis.
(c) k is zero.	**C.** The graph of $f(x)$ intersects the x-axis twice.

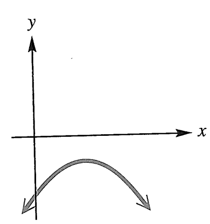
Concept Check *The following figures show several possible graphs of $f(x) = ax^2 + bx + c$. For the restrictions on a, b, and c given in Exercises 33–38, select the corresponding graph from choices A–F. (Hint: Use the discriminant.)*

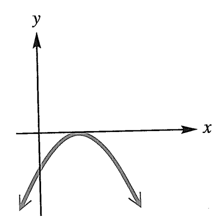
33. $a < 0$; $b^2 - 4ac = 0$
34. $a > 0$; $b^2 - 4ac < 0$
35. $a < 0$; $b^2 - 4ac < 0$
36. $a < 0$; $b^2 - 4ac > 0$
37. $a > 0$; $b^2 - 4ac > 0$
38. $a > 0$; $b^2 - 4ac = 0$

A.


B.


C.


D.


E.


F.