Vraag 2 (20 punte) | **Question 2** *(20 marks)*

In hierdie vraag bepaal jy die oplossing van die golfvergelyking,

In this question you solve the wave equation,

$$u_{tt} - a^2 u_{xx} = 0, \quad -\infty < x < \infty.$$

(a) Die golfvergelyking kan, met die verandering van veranderlikes $\xi = x - at$ en $\eta = x + ct$, na die differensiaalvergelyking $u_{\xi\eta} = 0$ gereduseer word. Toon hoe dit lei tot d'Alembert se vorm van die oplossing $u(x,t)$, dws

(a) *The wave equation can be reduced to the differential equation $u_{\xi\eta} = 0$ by the change of variables $\xi = x - at$ and $\eta = x + ct$. Show how this leads to the d'Alembert form of the solution $u(x,t)$, i.e.*

$$u(x,t) = \phi(x - at) + \psi(x + at),$$

waar ϕ en ψ willekeurige funksies is.

where ϕ and ψ are arbitrary functions.

(b) Indien dit gegee word dat

(b) *If it is given that*

$$u(x,0) = f(x), \qquad u_t(x,0) = 0, \quad -\infty < x < \infty,$$

toon aan dat

show that

$$\begin{aligned} \phi(x) + \psi(x) &= f(x), \\ -\phi'(x) + \psi'(x) &= 0. \end{aligned}$$

(c) Met die beginwaardes soos in (b), toon aan dat

(c) *Using the initial conditions in (b), show that*

$$u(x,t) = \frac{1}{2}\Big[f(x - at) + f(x + at)\Big].$$

(d) Indien die funksie $f(x)$ gegee word as

(d) *If the function $f(x)$ is given as*

$$f(x) = \begin{cases} 2, & -1 < x < 1 \\ 0, & \text{otherwise,} \end{cases}$$

toon aan dat

show that

$$f(x - at) = \begin{cases} 2, & -1 + at < x < 1 + at \\ 0, & \text{otherwise.} \end{cases}$$

Bereken ook vir $f(x + at)$.

Also calculate $f(x + at)$.

(e) Stip die oplossing $u(x,t)$ by

(e) *Plot the solution $u(x,t)$ at*

$$t = 0, \;\; t = \frac{1}{2a}, \;\; t = \frac{1}{a}, \;\; t = \frac{2}{a}.$$