59. $\left[-2, \frac{3}{2}\right] \cup [3, \infty)$
60. $[-5, -2] \cup \left[\frac{4}{3}, \infty\right)$
61. $(-\infty, -2] \cup [0, 2]$
62. $(-\infty, -4] \cup [0, 4]$
63. $(-\infty, -1) \cup (-1, 3)$
64. $(-\infty, -1)$
65. $[-4, -3] \cup [3, \infty)$
66. $(-\infty, -4] \cup [-3, 4]$
67. $(-\infty, \infty)$ 68. $\emptyset$ 69. $(-5, 3]$
70. $(-\infty, -1) \cup (4, \infty)$
71. $(-\infty, -2)$ 72. $(-2, 2)$
73. $(-\infty, 6) \cup \left[\frac{15}{2}, \infty\right)$
74. $(-\infty, 2) \cup (5, \infty)$
75. $(-\infty, 1) \cup \left(\frac{9}{5}, \infty\right)$
76. $\left(-\infty, \frac{2}{3}\right] \cup \left(\frac{5}{3}, \infty\right)$
77. $\left(-\infty, -\frac{3}{2}\right) \cup \left[-\frac{1}{2}, \infty\right)$
78. $\left(-2, -\frac{5}{3}\right]$ 79. $(-2, \infty)$
0. $(-\infty, -1)$
1. $\left(0, \frac{4}{11}\right) \cup \left(\frac{1}{2}, \infty\right)$
$\left(-\infty, -\frac{2}{3}\right) \cup \left[-\frac{1}{2}, 0\right)$
$(-\infty, -2] \cup (1, 2)$
$(-\infty, -5) \cup (-3, -1)$
$(-\infty, 5)$
$\left(-\infty, -\frac{3}{2}\right) \cup \left[-\frac{13}{9}, \infty\right)$
$\left[\frac{3}{2}, \infty\right)$ 88. $\left(-\infty, \frac{8}{9}\right)$
$\left(\frac{5}{2}, \infty\right)$ 90. $\left(-\infty, \frac{3}{5}\right]$
$\left[-\frac{8}{3}, \frac{3}{2}\right] \cup (6, \infty)$
$\left(-\frac{7}{2}, \frac{11}{9}\right) \cup \left(\frac{8}{3}, \infty\right)$
98 and 1999; 2001 and
It agrees favorably, except
year 2001.

Use the technique described in Relating Concepts Exercises 55–58 to solve each inequality. Write each solution set in interval notation.

59. $(2x - 3)(x + 2)(x - 3) \geq 0$

60. $(x + 5)(3x - 4)(x + 2) \geq 0$

61. $4x - x^3 \geq 0$

62. $16x - x^3 \geq 0$

63. $(x + 1)^2(x - 3) < 0$

64. $(x - 5)^2(x + 1) < 0$

65. $x^3 + 4x^2 - 9x - 36 \geq 0$

66. $x^3 + 3x^2 - 16x - 48 \leq 0$

67. $x^2(x + 4)^2 \geq 0$

68. $x^2(2x - 3)^2 < 0$

Solve each rational inequality. Write each solution set in interval notation. See Examples 8 and 9.

69. $\frac{x - 3}{x + 5} \leq 0$

70. $\frac{x + 1}{x - 4} > 0$

71. $\frac{x - 1}{x + 2} > 1$

72. $\frac{x - 6}{x + 2} < -1$

73. $\frac{3}{x - 6} \leq 2$

74. $\frac{3}{x - 2} < 1$

75. $\frac{4}{x - 1} < 5$

76. $\frac{6}{5 - 3x} \leq 2$

77. $\frac{10}{3 + 2x} \leq 5$

78. $\frac{1}{x + 2} \geq 3$

79. $\frac{7}{x + 2} \geq \frac{1}{x + 2}$

80. $\frac{5}{x + 1} > \frac{12}{x + 1}$

81. $\frac{3}{2x - 1} > \frac{-4}{x}$

82. $\frac{-5}{3x + 2} \geq \frac{5}{x}$

83. $\frac{4}{x - 2} \leq \frac{3}{x - 1}$

84. $\frac{4}{x + 1} < \frac{2}{x + 3}$

85. $\frac{x + 3}{x - 5} \leq 1$

86. $\frac{x + 2}{3 + 2x} \leq 5$

Solve each rational inequality. Write each solution set in interval notation.

87. $\frac{2x - 3}{x^2 + 1} \geq 0$

88. $\frac{9x - 8}{4x^2 + 25} < 0$

89. $\frac{(3x - 5)^2}{(2x - 5)^3} > 0$

90. $\frac{(5x - 3)^3}{(8x - 25)^2} \leq 0$

91. $\frac{(2x - 3)(3x + 8)}{(x - 6)^3} \geq 0$

92. $\frac{(9x - 11)(2x + 7)}{(3x - 8)^3} > 0$

(Modeling) Solve each problem. For Exercises 95 and 96, see Example 7.

93. ***Box Office Receipts*** U.S. box office receipts, in billions of dollars, for films in the years 1998 through 2002 are shown in the table.

Year	Receipts
1998	6.9
1999	7.4
2000	7.7
2001	8.4
2002	9.2

Source: Motion Picture Association of America.

4. 1996; 1996, 1997, and 1998
5. between (and inclusive of) 4 sec and 9.75 sec
6. between -1.5 sec and 4 sec
7. (a) $2.08 \times 10^{-5} \leq \frac{R}{72} \leq 8.33 \times 10^{-5}$
(b) between 5400 and 21,700

The receipts R are approximated reasonably well by the linear model

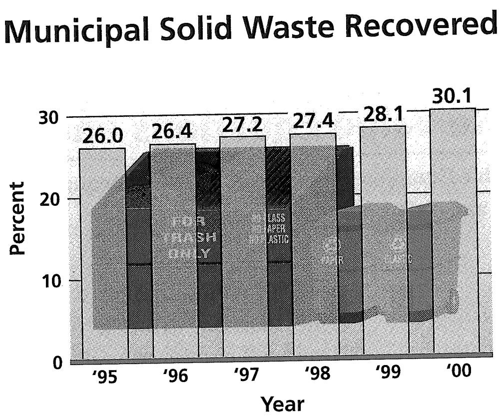
$$R = .56x + 6.8$$

where $x = 0$ corresponds to 1998, $x = 1$ corresponds to 1999, and so on. From the model, in what years was the revenue below \$7.5 billion? In what years did it exceed \$8.5 billion? Compare your answers with the data from the table.

94. *Recovery of Solid Waste* The percent W of municipal solid waste recovered is shown in the bar graph. The linear model

$$W = .737x + 25.7,$$

where $x = 0$ represents 1995, $x = 1$ represents 1996, and so on, fits the data reasonably well. Based on this model, when did the percent of waste recovered first exceed 26%? In what years was it between 26% and 28%?



Source: Municipal Solid Waste in the United States: 2000 Facts and Figures; U.S. Environmental Protection Agency.

95. *Height of a Projectile* A projectile is fired straight up from ground level. After t seconds, its height above the ground is s feet, where

$$s = -16t^2 + 220t.$$

For what time period is the projectile at least 624 ft above the ground?

96. *Velocity of an Object* Suppose the velocity of an object is given by

$$v = 2t^2 - 5t - 12,$$

where t is time in seconds. (Here t can be positive or negative.) Find the intervals where the velocity is negative.

97. *Cancer Risk from Radon Gas Exposure* Radon gas occurs naturally in homes and is produced when uranium radioactively decays into lead. Exposure to radon gas is a known lung cancer risk. According to the Environmental Protection Agency (EPA) the individual lifetime excess cancer risk R for radon exposure is between

$$1.5 \times 10^{-3} \quad \text{and} \quad 6.0 \times 10^{-3},$$

where $R = .01$ represents a 1% increase in risk of developing lung cancer.

(a) Calculate the range of individual annual risk by dividing R by an average life expectancy of 72 yr.

(b) Approximate the range of new cases of lung cancer each year (to the nearest hundred) caused by radon if the population of the United States is 260 million.

(Source: Indoor-Air Assessment: A Review of Indoor Air Quality Risk Characterization Studies. Report No. EPA/600/8-90/044, Environmental Protection Agency, 1991.)

100. $k < -4\sqrt{2}$ or $k > 4\sqrt{2}$

98. A student attempted to solve the inequality

$$\frac{2x-1}{x+2} \le 0$$

by multiplying both sides by $x + 2$ to get

$$2x - 1 \le 0$$
$$x \le \frac{1}{2}.$$

He wrote the solution set as $\left(-\infty, \frac{1}{2}\right]$. Is his solution correct? Explain.

99. A student solved the inequality $x^2 \le 16$ by taking the square root of both sides to get $x \le 4$. She wrote the solution set as $(-\infty, 4]$. Is her solution correct? Explain.

100. *Concept Check* Use the discriminant to find the values of k for which $x^2 - kx + 8 = 0$ has two real solutions.

1.8 Absolute Value Equations and Inequalities

Absolute Value Equations ▪ Absolute Value Inequalities ▪ Special Cases ▪ Absolute Value Models for Distance and Tolerance

Recall from Chapter R that the absolute value of a number a, written $|a|$, gives the distance from a to 0 on a number line. By this definition, the equation $|x| = 3$ can be solved by finding all real numbers at a distance of 3 units from 0. As shown in Figure 19, two numbers satisfy this equation, 3 and -3, so the solution set is $\{-3, 3\}$.

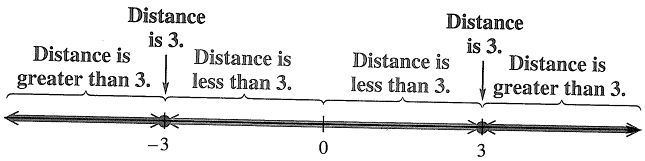


Figure 19

Similarly, $|x| < 3$ is satisfied by all real numbers whose distances from 0 are less than 3, that is, the interval $-3 < x < 3$ or $(-3, 3)$. See Figure 19. Finally, $|x| > 3$ is satisfied by all real numbers whose distances from 0 are greater than 3. As Figure 19 shows, these numbers are less than -3 or greater than 3, so the solution set is $(-\infty, -3) \cup (3, \infty)$. Notice in Figure 19 that the union of the solution sets of $|x| = 3$, $|x| < 3$, and $|x| > 3$ is the set of real numbers.

These observations support the following properties of absolute value.

TEACHING TIP Students will sometimes write Property 4 as $b < a < -b$. Caution them against this.

Properties of Absolute Value

1. For $b > 0$, $|a| = b$ **if and only if** $a = b$ **or** $a = -b$.
2. $|a| = |b|$ **if and only if** $a = b$ **or** $a = -b$.

For any positive number b:

3. $|a| < b$ **if and only if** $-b < a < b$.
4. $|a| > b$ **if and only if** $a < -b$ **or** $a > b$.