1.8 Exercises

1. F   **2.** B   **3.** D   **4.** E   **5.** G
6. A   **7.** C   **8.** H

9. $\left\{-\dfrac{1}{3}, 1\right\}$ **10.** $\left\{-\dfrac{7}{4}, \dfrac{3}{4}\right\}$

11. $\left\{\dfrac{2}{3}, \dfrac{8}{3}\right\}$ **12.** $\left\{\dfrac{4}{3}, \dfrac{10}{3}\right\}$

13. $\{-6, 14\}$ **14.** $\{-16, 12\}$

15. $\left\{\dfrac{5}{2}, \dfrac{7}{2}\right\}$ **16.** $\left\{\dfrac{1}{8}, \dfrac{7}{8}\right\}$

17. $\left\{-\dfrac{4}{3}, \dfrac{2}{9}\right\}$ **18.** $\left\{\dfrac{1}{5}, 7\right\}$

19. $\left\{-\dfrac{7}{3}, -\dfrac{1}{7}\right\}$ **20.** $\{0, 1\}$

21. $\{1\}$ **22.** $\{2\}$ **23.** $(-\infty, \infty)$

26. (a) $\{0\}$ (b) $(-\infty, \infty)$
(c) $\{-1, 0, 1\}$ (d) $\emptyset$

27. $(-4, -1)$ **28.** $\left(\dfrac{2}{3}, 2\right)$

29. $(-\infty, -4] \cup [-1, \infty)$

30. $\left(-\infty, \dfrac{2}{3}\right] \cup [2, \infty)$

31. $\left(-\dfrac{3}{2}, \dfrac{5}{2}\right)$ **32.** $\left(-\dfrac{8}{5}, \dfrac{2}{5}\right)$

33. $(-\infty, 0) \cup (6, \infty)$
34. $(-\infty, -3) \cup (1, \infty)$

35. $\left(-\infty, -\dfrac{2}{3}\right) \cup (4, \infty)$

36. $(-\infty, 1) \cup \left(\dfrac{11}{3}, \infty\right)$

37. $\left[-\dfrac{2}{3}, 4\right]$ **38.** $\left[1, \dfrac{11}{3}\right]$

39. $\left[-1, -\dfrac{1}{2}\right]$

40. $\left(-\infty, \dfrac{26}{9}\right) \cup \left(\dfrac{34}{9}, \infty\right)$

Concept Check   *Match each equation or inequality in Column I with the graph of its solution set in Column II.*

I

1. $|x| = 4$

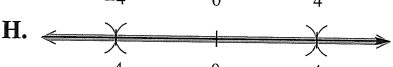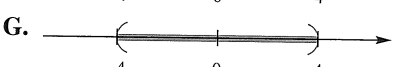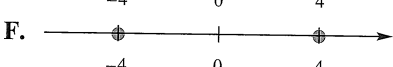**2.** $|x| = -4$

3. $|x| > -4$

4. $|x| > 4$

5. $|x| < 4$

6. $|x| \geq 4$

7. $|x| \leq 4$

8. $|x| \neq 4$

II

A.

B.

C.

D.

E.

F.

G.

H.
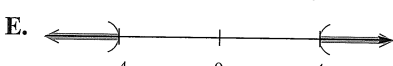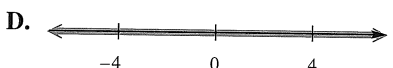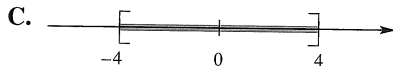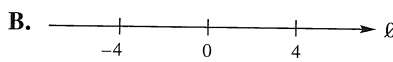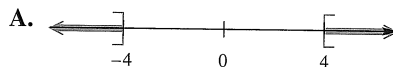
Solve each equation. See Example 1.

9. $|3x - 1| = 2$ **10.** $|4x + 2| = 5$ **11.** $|5 - 3x| = 3$

12. $|7 - 3x| = 3$ **13.** $\left|\dfrac{x - 4}{2}\right| = 5$ **14.** $\left|\dfrac{x + 2}{2}\right| = 7$

15. $\left|\dfrac{5}{x - 3}\right| = 10$ **16.** $\left|\dfrac{3}{2x - 1}\right| = 4$ **17.** $\left|\dfrac{6x + 1}{x - 1}\right| = 3$

18. $\left|\dfrac{3x - 4}{2x + 3}\right| = 1$ **19.** $|2x - 3| = |5x + 4|$ **20.** $|x + 1| = |3x - 1|$

21. $|4 - 3x| = |2 - 3x|$ **22.** $|3 - 2x| = |5 - 2x|$ **23.** $|5x - 2| = |2 - 5x|$

24. The equation $|5x - 6| = 6x$ cannot have a negative solution. Why?

25. The equation $|7x + 3| = -7x$ cannot have a positive solution. Why?

26. *Concept Check*   Determine the solution set of each equation by inspection.
　　(a) $-|x| = |x|$ (b) $|-x| = |x|$ (c) $|x^2| = |x|$ (d) $-|x| = 3$

Solve each inequality. Give the solution set using interval notation. See Example 2.

27. $|2x + 5| < 3$ **28.** $|3x - 4| < 2$ **29.** $|2x + 5| \geq 3$

30. $|3x - 4| \geq 2$ **31.** $\left|x - \dfrac{1}{2}\right| < 2$ **32.** $\left|x + \dfrac{3}{5}\right| < 1$

33. $4|x - 3| > 12$ **34.** $3|x + 1| > 6$ **35.** $|5 - 3x| > 7$

36. $|7 - 3x| > 4$ **37.** $|5 - 3x| \leq 7$ **38.** $|7 - 3x| \leq 4$

39. $\left|\dfrac{2}{3}x + \dfrac{1}{2}\right| \leq \dfrac{1}{6}$ **40.** $\left|\dfrac{5}{3} - \dfrac{1}{2}x\right| > \dfrac{2}{9}$

Left margin answers (partially cut off):

$-\frac{1}{2}\}$ **42.** $\left\{\frac{7}{3},3\right\}$

44. $\left\{\frac{1}{2},\frac{3}{2}\right\}$

$\frac{2}{3}\right)$ **46.** $\left(-\frac{7}{5},\frac{3}{5}\right)$

$\frac{13}{10}\right)$ **48.** $\left(-\frac{5}{3},\frac{4}{3}\right)$

$\frac{3}{2}\right] \cup \left[\frac{7}{2},\infty\right)$

$] \cup [3,\infty)$

$)$ **52.** $(-\infty,\infty)$

$\varnothing$ **55.** $\left\{-\frac{5}{8}\right\}$

57. $\varnothing$ **58.** $\varnothing$

60. $\left\{-\frac{2}{3}\right\}$

$-\frac{2}{3}\right) \cup \left(-\frac{2}{3},\infty\right)$

$-\frac{3}{4}\right) \cup \left(-\frac{3}{4},\infty\right)$

6

$x = 6; \{-2,3\}$

$x = -6; \left\{\frac{1}{2} \pm \frac{\sqrt{23}}{2}i\right\}$

$3, \frac{1}{2} \pm \frac{\sqrt{23}}{2}i\right\}$

$6\}$

$-\frac{1}{3},0\right\}$ **69.** $\{-1,1\}$

$-\frac{2}{3},\frac{2}{3},3\right\}$

$.$ $(-\infty,\infty)$

$4) \cup (4,\infty)$

$9) \cup (9,\infty)$

ses 75–82, the
in absolute value bars
placed by its additive
or example, in
75, $p - q$ may be
$- p$.
$q| = 5$
$s| = 9$

Solve each equation or inequality. See Example 3.

41. $|4x + 3| - 2 = -1$ **42.** $|8 - 3x| - 3 = -2$ **43.** $|6 - 2x| + 1 = 3$

44. $|4 - 4x| + 2 = 4$ **45.** $|3x + 1| - 1 < 2$ **46.** $|5x + 2| - 2 < 3$

47. $\left|5x + \frac{1}{2}\right| - 2 < 5$ **48.** $\left|2x + \frac{1}{3}\right| + 1 < 4$ **49.** $|10 - 4x| + 1 \geq 5$

50. $|12 - 6x| + 3 \geq 9$

Solve each equation or inequality. See Example 4.

51. $|10 - 4x| \geq -3$ **52.** $|12 - 9x| \geq -6$ **53.** $|6 - 3x| < -5$

54. $|18 - 3x| < -12$ **55.** $|8x + 5| = 0$ **56.** $|7 + 2x| = 0$

57. $|4.3x + 8.6| < 0$ **58.** $|1.5x - 3| < 0$ **59.** $|2x + 1| \leq 0$

60. $|3x + 2| \leq 0$ **61.** $|3x + 2| > 0$ **62.** $|4x + 3| > 0$

Relating Concepts

For individual or collaborative investigation
(Exercises 63–66)

To see how to solve an equation that involves the absolute value of a quadratic polynomial, such as $|x^2 - x| = 6$, **work Exercises 63–66 in order.**

63. For $x^2 - x$ to have an absolute value equal to 6, what are the two possible values that it may be? (*Hint:* One is positive and the other is negative.)

64. Write an equation stating that $x^2 - x$ is equal to the positive value you found in Exercise 63, and solve it using factoring.

65. Write an equation stating that $x^2 - x$ is equal to the negative value you found in Exercise 63, and solve it using the quadratic formula. (*Hint:* The solutions are not real numbers.)

66. Give the complete solution set of $|x^2 - x| = 6$, using the results from Exercises 64 and 65.

Use the method described in Relating Concepts Exercises 63–66 to solve each equation or inequality.

67. $|4x^2 - 23x - 6| = 0$ **68.** $|6x^3 + 23x^2 + 7x| = 0$

69. $|x^2 + 1| = |2x|$ **70.** $\left|\dfrac{x^2 + 2}{x}\right| = \dfrac{11}{3}$

71. $|x^4 + 2x^2 + 1| < 0$ **72.** $|x^4 + 2x^2 + 1| \geq 0$

73. $\left|\dfrac{3x + 1}{x - 4}\right| \geq 0$ **74.** $\left|\dfrac{8x + 7}{x - 9}\right| \geq 0$

75. *Concept Check*   Write an equation involving absolute value that says the distance between p and q is 5 units.

76. *Concept Check*   Write an equation involving absolute value that says the distance between r and s is 9 units.

77. $|m - 9| \le 8$

78. $|z - 12| \ge 2$

79. $|p - 9| < .0001$

80. $|k - 3| < .0002$

81. $|r - 19| \ge 1$

82. $|q - 12| \le 4$

83. $(.9996, 1.0004)$

84. $(.3998, .4002)$ **85.** $[6.5, 9.5]$

86. $[-140, -28]$

87. $|F - 730| \le 50$

88. $|x - 123| \le 25$;
$|x - 21| \le 5$

89. $25.33 \le R_L \le 28.17$;
$36.58 \le R_E \le 40.92$

90. $5699.25 \le T_L \le 6338.25$;
$8230.5 \le T_E \le 9207$

Write each statement as an absolute value equation or inequality. See Example 5.

77. m is no more than 8 units from 9.

78. z is no less than 2 units from 12.

79. p is within .0001 unit of 9.

80. k is within .0002 unit of 3.

81. r is no less than 1 unit from 19.

82. q is no more than 4 units from 12.

83. *Tolerance*   Suppose that $y = 5x + 1$ and we want y to be within .002 unit of 6. For what values of x will this be true?

84. *Tolerance*   Repeat Exercise 83, but let $y = 10x + 2$.

(Modeling)   Solve each problem. See Example 6.

85. *Weights of Babies*   Dr. Tydings has found that, over the years, 95% of the babies he has delivered weighed y pounds, where

$$|y - 8.0| \le 1.5.$$

What range of weights corresponds to this inequality?

86. *Temperatures on Mars*   The temperatures on the surface of Mars in degrees Celsius approximately satisfy the inequality

$$|C + 84| \le 56.$$

What range of temperatures corresponds to this inequality?

87. *Conversion of Methanol to Gasoline*   The industrial process that is used to convert methanol to gasoline is carried out at a temperature range of 680°F to 780°F. Using F as the variable, write an absolute value inequality that corresponds to this range.

88. *Wind Power Extraction Tests*   When a model kite was flown in crosswinds in tests to determine its limits of power extraction, it attained speeds of 98 to 148 ft per sec in winds of 16 to 26 ft per sec. Using x as the variable in each case, write absolute value inequalities that correspond to these ranges.

(Modeling) Carbon Dioxide Emissions   When humans breathe, carbon dioxide is emitted. In one study, the emission rates of carbon dioxide by college students were measured during both lectures and exams. The average individual rate R_L (in grams per hour) during a lecture class satisfied the inequality

$$|R_L - 26.75| \le 1.42,$$

whereas during an exam the rate R_E satisfied the inequality

$$|R_E - 38.75| \le 2.17.$$

(*Source:* Wang, T. C., *ASHRAE Trans.*, 81 (Part 1), 32, 1975.)

Use this information in Exercises 89–91.

89. Find the range of values for R_L and R_E.

90. The class had 225 students. If T_L and T_E represent the total amounts of carbon dioxide in grams emitted during a one-hour lecture and exam, respectively, write inequalities that model the ranges for T_L and T_E.

91. Discuss any reasons that might account for these differences between the rates during lectures and the rates during exams.

92. Is $|a - b|^2$ always equal to $(b - a)^2$? Explain your answer.