

## 1.6 Exercises

1.  $-\frac{3}{2}, 6$  2.  $-1$  3.  $2, -1$   
 4.  $-3, 1$  5.  $0$  6.  $0$  7.  $\{-10\}$   
 8.  $\left\{\frac{3}{5}\right\}$  9.  $\emptyset$  10.  $\emptyset$   
 11.  $\emptyset$  12.  $\emptyset$  13.  $\{-9\}$   
 14.  $\{3\}$  15.  $\{-2\}$   
 16.  $\left\{-\frac{1}{4}\right\}$  17.  $\emptyset$  18.  $\emptyset$   
 19.  $\left\{-\frac{5}{2}, \frac{1}{9}\right\}$  20.  $\left\{-\frac{1}{3}, \frac{7}{2}\right\}$   
 21.  $\left\{\frac{3}{4}, 1\right\}$  22.  $\left\{\frac{9}{4}, \frac{4}{3}\right\}$   
 23.  $\{3, 5\}$  24.  $\left\{-\frac{3}{2}, 4\right\}$   
 25.  $\left\{-2, \frac{5}{4}\right\}$  26.  $\left\{-\frac{1}{3}, \frac{8}{3}\right\}$   
 27.  $\{3\}$  28.  $\{6\}$  29.  $\{-1\}$   
 30.  $\{3\}$  31.  $\{5\}$  32.  $\{-1, 3\}$   
 33.  $\{9\}$  34.  $\{8\}$  35.  $\{9\}$   
 36.  $\{16\}$  37.  $\emptyset$  38.  $\left\{\frac{5}{4}\right\}$

Decide what values of the variable cannot possibly be solutions for each equation. Do not solve. See Examples 1 and 2.

1.  $\frac{5}{2x+3} + \frac{1}{x-6} = 0$  2.  $\frac{2}{x+1} - \frac{3}{5x+5} = 0$   
 3.  $\frac{3}{x-2} + \frac{1}{x+1} = \frac{1}{x^2-x-2}$  4.  $\frac{2}{x+3} - \frac{5}{x-1} = \frac{-1}{x^2+2x-3}$   
 5.  $\frac{1}{4x} + \frac{2}{x} = 3$  6.  $\frac{5}{2x} - \frac{2}{x} = 6$

Solve each equation. See Example 1.

7.  $\frac{2x+5}{2} - \frac{3x}{x-2} = x$  8.  $\frac{4x+3}{4} - \frac{2x}{x+1} = x$   
 9.  $\frac{x}{x-3} = \frac{3}{x-3} + 3$  10.  $\frac{x}{x-4} = \frac{4}{x-4} + 4$   
 11.  $\frac{2}{x-3} - \frac{3}{x+3} = \frac{12}{x^2-9}$  12.  $\frac{3}{x-2} + \frac{1}{x+2} = \frac{12}{x^2-4}$   
 13.  $\frac{4}{x^2+x-6} - \frac{1}{x^2-4} = \frac{2}{x^2+5x+6}$   
 14.  $\frac{3}{x^2+x-2} - \frac{1}{x^2-1} = \frac{7}{2x^2+6x+4}$

Solve each equation. See Example 2.

15.  $\frac{2x+1}{x-2} + \frac{3}{x} = \frac{-6}{x^2-2x}$  16.  $\frac{4x+3}{x+1} + \frac{2}{x} = \frac{1}{x^2+x}$   
 17.  $\frac{-x}{x-1} + \frac{1}{x+1} = \frac{-2}{x^2-1}$  18.  $\frac{x}{x+1} + \frac{1}{x-1} = \frac{2}{x^2-1}$   
 19.  $\frac{5}{x^2} - \frac{43}{x} = 18$  20.  $\frac{7}{x^2} + \frac{19}{x} = 6$   
 21.  $2 = \frac{3}{2x-1} + \frac{-1}{(2x-1)^2}$  22.  $6 = \frac{7}{2x-3} + \frac{3}{(2x-3)^2}$   
 23.  $\frac{2x-5}{x} = \frac{x-2}{3}$  24.  $\frac{x+4}{2x} = \frac{x-1}{3}$   
 25.  $\frac{2x}{x-2} = 5 + \frac{4x^2}{x-2}$  26.  $\frac{-3x}{2} + \frac{9x-5}{3} = \frac{11x+8}{6x}$

Solve each equation. See Examples 3–5 and 8.

27.  $x - \sqrt{2x+3} = 0$  28.  $x - \sqrt{3x+18} = 0$   
 29.  $\sqrt{3x+7} = 3x+5$  30.  $\sqrt{4x+13} = 2x-1$   
 31.  $\sqrt{4x+5} - 2 = 2x-7$  32.  $\sqrt{6x+7} - 1 = x+1$   
 33.  $\sqrt{4x} - x + 3 = 0$  34.  $\sqrt{2x} - x + 4 = 0$   
 35.  $\sqrt{x} - \sqrt{x-5} = 1$  36.  $\sqrt{x} - \sqrt{x-12} = 2$   
 37.  $\sqrt{x+7} + 3 = \sqrt{x-4}$  38.  $\sqrt{x+5} - 2 = \sqrt{x-1}$

39.  $\{\pm 2\}$  40.  $\left\{\frac{10}{9}, 2\right\}$   
 41.  $\{0, 3\}$  42.  $\{8\}$  43.  $\{-2\}$   
 44.  $\{27\}$  45.  $\left\{-\frac{2}{9}, 2\right\}$   
 46.  $\{3\}$  47.  $\{4\}$  48.  $\{0, 9\}$   
 49.  $\{-2\}$  50.  $\left\{-\frac{2}{3}\right\}$   
 51.  $\left\{\frac{2}{5}, 1\right\}$  52.  $\left\{\frac{4}{3}, 2\right\}$   
 53.  $\left\{\frac{3}{2}\right\}$  54.  $\{5\}$  55.  $\{31\}$   
 56.  $\{0\}$  57.  $\{-3, 1\}$   
 58.  $\{-8, 2\}$  59.  $\{-27, 3\}$   
 60.  $\left\{-\frac{64}{3}, 4\right\}$   
 61.  $\left\{\pm 1, \pm \frac{\sqrt{10}}{2}\right\}$   
 62.  $\left\{\pm \frac{\sqrt{6}}{2}, \pm \frac{\sqrt{2}}{2}\right\}$   
 63.  $\{\pm\sqrt{3}, \pm i\sqrt{5}\}$   
 64.  $\left\{\pm \frac{\sqrt{15}}{3}, \pm i\sqrt{5}\right\}$   
 65.  $\left\{\frac{1}{4}, 1\right\}$  66.  $\{1, 9\}$   
 67.  $\{0, 8\}$  68.  $\left\{0, \frac{1}{81}\right\}$   
 69.  $\{-63, 28\}$  70.  $\{-13, 1\}$   
 71.  $\{0, 31\}$  72.  $\{-13, 3\}$   
 73.  $\left\{\frac{-6 \pm 2\sqrt{3}}{3}, \frac{-4 \pm \sqrt{2}}{2}\right\}$   
 74.  $\left\{\frac{8 \pm \sqrt{2}}{2}, \frac{8 \pm \sqrt{3}}{2}\right\}$   
 75.  $\left\{-\frac{2}{7}, 5\right\}$  76.  $\left\{-\frac{7}{4}, \frac{1}{2}\right\}$   
 77.  $\left\{-\frac{1}{27}, \frac{1}{8}\right\}$   
 78.  $\{-1, 1, \pm 2i\sqrt{2}\}$   
 81.  $\{16\}$ ;  $u = -3$  does not lead to a solution of the equation.  
 82.  $\{16\}$ ; 9 does not satisfy the equation. 84.  $\{4\}$

39.  $\sqrt{x+2} = \sqrt{2x+5} - 1$

41.  $\sqrt{3x} = \sqrt{5x+1} - 1$

43.  $\sqrt{x+2} = 1 - \sqrt{3x+7}$

45.  $\sqrt{2\sqrt{7x+2}} = \sqrt{3x+2}$

47.  $3 - \sqrt{x} = \sqrt{2\sqrt{x}-3}$

49.  $\sqrt[3]{4x+3} = \sqrt[3]{2x-1}$

51.  $\sqrt[3]{5x^2-6x+2} - \sqrt[3]{x} = 0$

53.  $(2x+5)^{1/3} - (6x-1)^{1/3} = 0$

55.  $\sqrt{x-15} = 2$

57.  $\sqrt{x^2+2x} = \sqrt[3]{3}$

59.  $(x^2+24x)^{1/4} = 3$

40.  $\sqrt{4x+1} = \sqrt{x-1} + 2$

42.  $\sqrt{2x} = \sqrt{3x+12} - 2$

44.  $\sqrt{2x-5} = 2 + \sqrt{x-2}$

46.  $\sqrt{3\sqrt{2x+3}} = \sqrt{5x-6}$

48.  $\sqrt{x+2} = \sqrt{4+7\sqrt{x}}$

50.  $\sqrt[3]{2x} = \sqrt[3]{5x+2}$

52.  $\sqrt[3]{3x^2-9x+8} = \sqrt[3]{x}$

54.  $(3x+7)^{1/3} - (4x+2)^{1/3} = 0$

56.  $\sqrt[3]{3x+1} = 1$

58.  $\sqrt{x^2+6x} = 2$

60.  $(3x^2+52x)^{1/4} = 4$

Solve each equation. See Examples 6 and 7.

61.  $2x^4 - 7x^2 + 5 = 0$

63.  $x^4 + 2x^2 - 15 = 0$

65.  $(2x-1)^{2/3} = x^{1/3}$

67.  $x^{2/3} = 2x^{1/3}$

69.  $(x-1)^{2/3} + (x-1)^{1/3} - 12 = 0$

71.  $(x+1)^{2/5} - 3(x+1)^{1/5} + 2 = 0$

73.  $6(x+2)^4 - 11(x+2)^2 = -4$

75.  $10x^{-2} + 33x^{-1} - 7 = 0$

77.  $x^{-2/3} + x^{-1/3} - 6 = 0$

62.  $4x^4 - 8x^2 + 3 = 0$

64.  $3x^4 + 10x^2 - 25 = 0$

66.  $(x-3)^{2/5} = (4x)^{1/5}$

68.  $3x^{3/4} = x^{1/2}$

70.  $(2x-1)^{2/3} + 2(2x-1)^{1/3} - 3 = 0$

72.  $(x+5)^{4/3} + (x+5)^{2/3} - 20 = 0$

74.  $8(x-4)^4 - 10(x-4)^2 = -3$

76.  $7x^{-2} - 10x^{-1} - 8 = 0$

78.  $2x^{-4/3} - x^{-2/3} - 1 = 0$

79. Refer to the equation in Exercise 39. A student attempted to solve the equation by “squaring both sides” to get  $x+2 = (2x+5) + 1$ . What was incorrect about the student’s method?
80. Refer to the equation in Exercise 48. What should be the first step in solving the equation using the method described in this section?

## Relating Concepts

For individual or collaborative investigation

(Exercises 81–84)

In this section we introduced methods of solving equations quadratic in form by substitution, and solving equations involving radicals by raising both sides of the equation to a power. Suppose we wish to solve

$$x - \sqrt{x} - 12 = 0.$$

We can solve this equation using either of the two methods. **Work Exercises 81–84 in order, to see how both methods apply.**

81. Let  $u = \sqrt{x}$  and solve the equation by substitution. What is the value of  $u$  that does not lead to a solution of the equation?
82. Solve the equation by isolating  $\sqrt{x}$  on one side and then squaring. What is the value of  $x$  that does not satisfy the equation?
83. Which one of the methods used in Exercises 81 and 82 do you prefer? Why?
84. Solve  $3x - 2\sqrt{x} - 8 = 0$  using one of the two methods described.