Relating Concepts

For individual or collaborative investigation
(Exercises 7–10)

Work Exercises 7–10 in order.

7. Let $x = 4$ and $y = 2$. Evaluate **(a)** $\frac{1}{x} + \frac{1}{y}$ and **(b)** $\frac{1}{x+y}$.

8. Are the answers for Exercises 7(a) and (b) the same? What can you conclude?

9. Let $x = 3$ and $y = 5$. Evaluate **(a)** $\frac{1}{x} - \frac{1}{y}$ and **(b)** $\frac{1}{x-y}$.

10. Are the answers for Exercises 9(a) and (b) the same? What can you conclude?

Write each rational expression in lowest terms. See Example 1.

11. $\dfrac{8k+16}{9k+18}$ **12.** $\dfrac{20r+10}{30r+15}$ **13.** $\dfrac{3(3-t)}{(t+5)(t-3)}$ **14.** $\dfrac{-8(4-y)}{(y+2)(y-4)}$

15. $\dfrac{8x^2+16x}{4x^2}$ **16.** $\dfrac{36y^2+72y}{9y}$ **17.** $\dfrac{m^2-4m+4}{m^2+m-6}$ **18.** $\dfrac{r^2-r-6}{r^2+r-12}$

19. $\dfrac{8m^2+6m-9}{16m^2-9}$ **20.** $\dfrac{6y^2+11y+4}{3y^2+7y+4}$

Find each product or quotient. See Example 2.

21. $\dfrac{15p^3}{9p^2} \div \dfrac{6p}{10p^2}$ **22.** $\dfrac{3r^2}{9r^3} \div \dfrac{8r^3}{6r}$

23. $\dfrac{2k+8}{6} \div \dfrac{3k+12}{2}$ **24.** $\dfrac{5m+25}{10} \cdot \dfrac{12}{6m+30}$

25. $\dfrac{x^2+x}{5} \cdot \dfrac{25}{xy+y}$ **26.** $\dfrac{3m-15}{4m-20} \cdot \dfrac{m^2-10m+25}{12m-60}$

27. $\dfrac{4a+12}{2a-10} \div \dfrac{a^2-9}{a^2-a-20}$ **28.** $\dfrac{6r-18}{9r^2+6r-24} \cdot \dfrac{12r-16}{4r-12}$

29. $\dfrac{p^2-p-12}{p^2-2p-15} \cdot \dfrac{p^2-9p+20}{p^2-8p+16}$ **30.** $\dfrac{x^2+2x-15}{x^2+11x+30} \cdot \dfrac{x^2+2x-24}{x^2-8x+15}$

31. $\dfrac{m^2+3m+2}{m^2+5m+4} \div \dfrac{m^2+5m+6}{m^2+10m+24}$ **32.** $\dfrac{y^2+y-2}{y^2+3y-4} \div \dfrac{y^2+3y+2}{y^2+4y+3}$

33. $\dfrac{xz-xw+2yz-2yw}{z^2-w^2} \cdot \dfrac{4z+4w+xz+wx}{16-x^2}$

34. $\dfrac{ac+ad+bc+bd}{a^2-b^2} \cdot \dfrac{a^3-b^3}{2a^2+2ab+2b^2}$

35. $\dfrac{x^3+y^3}{x^3-y^3} \cdot \dfrac{x^2-y^2}{x^2+2xy+y^2}$ **36.** $\dfrac{x^2-y^2}{(x-y)^2} \cdot \dfrac{x^2-xy+y^2}{x^2-2xy+y^2} \div \dfrac{x^3+y^3}{(x-y)^4}$

37. *Concept Check* Which of the following rational expressions equals -1? (In parts A, B, and D, $x \neq -4$, and in part C, $x \neq 4$.) (*Hint:* There may be more than one answer.)

A. $\dfrac{x-4}{x+4}$ **B.** $\dfrac{-x-4}{x+4}$ **C.** $\dfrac{x-4}{4-x}$ **D.** $\dfrac{x-4}{-x-4}$

$\frac{19}{6k}$ **40.** $\frac{47}{20p}$ **41.** $\frac{137}{30m}$

$\frac{101}{12p}$ **43.** $\frac{a-b}{a^2}$

$\frac{3z+x}{z^2}$ **45.** $\frac{5-22x}{12x^2y}$

$\frac{7-4a^2b}{18a^3b^2}$ **47.** 3 **48.** 2

$\frac{2x}{(x+z)(x-z)}$

$\frac{2m^2+2}{m-1)(m+1)}$

$\frac{4}{-2}$ or $\frac{-4}{2-a}$

$\frac{2q}{-q}$ or $\frac{-2q}{q-p}$

$\frac{x+y}{x-y}$ or $\frac{-3x-y}{y-2x}$

$\frac{2m-6}{m-4}$ or $\frac{2m+6}{4-3m}$

$\frac{4x-7}{-x+1}$

$\frac{5x-18}{-2x+4}$

$\frac{2x^2-9x}{-3)(x+4)(x-4)}$

$\frac{p^2+8p}{+1)(p-5)(3p-2)}$

$\frac{-1}{1}$ **60.** $\frac{y-1}{y+1}$

$\frac{1}{1}$ **62.** $\frac{-3}{y+3}$

$-b)(1+b)$

$(1-b)$

$n-1)$

$+1$

$-4m-1$

$n-2$

$-16p+3$

$p+4$

$\frac{1}{h)}$

$\frac{-2x-h}{9)[(x+h)^2+9]}$

38. In your own words, explain how to find the least common denominator of several fractions.

Perform each addition or subtraction. See Example 3.

39. $\frac{3}{2k}+\frac{5}{3k}$

40. $\frac{8}{5p}+\frac{3}{4p}$

41. $\frac{1}{6m}+\frac{2}{5m}+\frac{4}{m}$

42. $\frac{8}{3p}+\frac{5}{4p}+\frac{9}{2p}$

43. $\frac{1}{a}-\frac{b}{a^2}$

44. $\frac{3}{z}+\frac{x}{z^2}$

45. $\frac{5}{12x^2y}-\frac{11}{6xy}$

46. $\frac{7}{18a^3b^2}-\frac{2}{9ab}$

47. $\frac{17y+3}{9y+7}-\frac{-10y-18}{9y+7}$

48. $\frac{7x+8}{3x+2}-\frac{x+4}{3x+2}$

49. $\frac{1}{x+z}+\frac{1}{x-z}$

50. $\frac{m+1}{m-1}+\frac{m-1}{m+1}$

51. $\frac{3}{a-2}-\frac{1}{2-a}$

52. $\frac{q}{p-q}-\frac{q}{q-p}$

53. $\frac{x+y}{2x-y}-\frac{2x}{y-2x}$

54. $\frac{m-4}{3m-4}+\frac{3m+2}{4-3m}$

55. $\frac{4}{x+1}+\frac{1}{x^2-x+1}-\frac{12}{x^3+1}$

56. $\frac{5}{x+2}+\frac{2}{x^2-2x+4}-\frac{60}{x^3+8}$

57. $\frac{3x}{x^2+x-12}-\frac{x}{x^2-16}$

58. $\frac{p}{2p^2-9p-5}-\frac{2p}{6p^2-p-2}$

Simplify each expression. See Example 4.

59. $\dfrac{1+\frac{1}{x}}{1-\frac{1}{x}}$

60. $\dfrac{2-\frac{2}{y}}{2+\frac{2}{y}}$

61. $\dfrac{\frac{1}{x+1}-\frac{1}{x}}{\frac{1}{x}}$

62. $\dfrac{\frac{1}{y+3}-\frac{1}{y}}{\frac{1}{y}}$

63. $\dfrac{1+\frac{1}{1-b}}{1-\frac{1}{1+b}}$

64. $m-\dfrac{m}{m+\frac{1}{2}}$

65. $\dfrac{m-\frac{1}{m^2-4}}{\frac{1}{m+2}}$

66. $\dfrac{\frac{3}{p^2-16}+p}{\frac{1}{p-4}}$

67. $\dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$

68. $\dfrac{\frac{1}{(x+h)^2+9}-\frac{1}{x^2+9}}{h}$

(Modeling) Distance from the Origin of the Nile River *The Nile River in Africa is about* 4000 mi *long. The Nile begins as an outlet of Lake Victoria at an altitude of* 7000 ft *above sea level and empties into the Mediterranean Sea at sea level* (0 ft). *The distance from its origin in thousands of miles is related to its height above sea level in thousands of feet* (x) *by the rational expression*

$$\frac{7-x}{.639x+1.75}.$$

2305 mi
and dollars)
sand dollars)

For example, when the river is at an altitude of 600 ft, $x = .6$ *(thousand), and the distance from the origin is*

$$\frac{7 - .6}{.639(.6) + 1.75} \approx 3,$$

which represents 3000 mi. *(Source: The World Almanac and Book of Facts,* 2001.*)*

69. What is the distance from the origin of the Nile when the river has an altitude of 7000 ft?

70. Find the distance from the origin of the Nile when the river is 1200 ft high.

(Modeling) Cost-Benefit Model for a Pollutant In situations involving environmental pollution, a cost-benefit model expresses cost in terms of the percentage of pollutant removed from the environment. Suppose a cost-benefit model is expressed as

$$y = \frac{6.7x}{100 - x},$$

where y is the cost in thousands of dollars of removing x percent of a certain pollutant. Find the value of y for each given value of x.

71. $x = 75$ (75%) **72.** $x = 95$ (95%)

ional Exponents

ve Exponents and the Quotient Rule ▪ Rational Exponents ▪ Complex Fractions Revisited

In this section we complete our review of exponents.

Negative Exponents and the Quotient Rule In the product rule, $a^m \cdot a^n = a^{m+n}$, the exponents are *added.* By the definition of exponent in Section R.1, if $a \neq 0$,

$$\frac{a^3}{a^7} = \frac{a \cdot a \cdot a}{a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a} = \frac{1}{a \cdot a \cdot a \cdot a} = \frac{1}{a^4}.$$

This example suggests that we should *subtract* exponents when dividing. Subtracting exponents gives

$$\frac{a^3}{a^7} = a^{3-7} = a^{-4}.$$

The only way to keep these results consistent is to define a^{-4} as $\frac{1}{a^4}$. This example suggests the following definition.

Negative Exponent

If a is a nonzero real number and n is any integer, then

$$a^{-n} = \frac{1}{a^n}.$$