

$$\begin{aligned}
 \text{(c)} \quad (2a - 1)^3 + 8 &= t^3 + 8 && \text{Let } 2a - 1 = t. \\
 &= t^3 + 2^3 && \text{Write as a sum of cubes.} \\
 &= (t + 2)(t^2 - 2t + 4) && \text{Factor.} \\
 &= [(2a - 1) + 2][(2a - 1)^2 - 2(2a - 1) + 4] \\
 &&& \text{Let } t = 2a - 1. \\
 &= (2a + 1)(4a^2 - 4a + 1 - 4a + 2 + 4) \\
 &&& \text{Add; multiply.} \\
 &= (2a + 1)(4a^2 - 8a + 7) && \text{Combine like terms.}
 \end{aligned}$$

Now try Exercises 71, 73, and 89.

ises

$3(5r - 9)$
 $9z(z^3 + 9)$
 $hj(5h + 1)$
 $y)$
 $w^2)$
 $- 3m)$
 $- 5rs^2)$
 $2m)$
 $5)$
 $5)$
 $2)$
 $- 7m + 7)$
 $+ 31a + 46)$
 $- 5)$
 $+ 7)$
 $a)$
 $r^2)$
 $+ p)$
 $ect.$
 $- 4)$
 $+ 3)$
 $+ 4)$
 $- 2)$
 $+ 3p)$
 $n + 3r)$
 $- 2b)$
 $- t)$
 $- m)$
 $3a + 2b)$
 $(2x + 5z)$
 $36. (4p - 5)^2$

Factor out the greatest common factor from each polynomial. See Examples 1 and 2.

- | | | |
|--|---|-----------------|
| 1. $12m + 60$ | 2. $15r - 27$ | 3. $8k^3 + 24k$ |
| 4. $9z^4 + 81z$ | 5. $xy - 5xy^2$ | 6. $5h^2j + hj$ |
| 7. $-4p^3q^4 - 2p^2q^5$ | 8. $-3z^5w^2 - 18z^3w^4$ | |
| 9. $4k^2m^3 + 8k^4m^3 - 12k^2m^4$ | 10. $28r^4s^2 + 7r^3s - 35r^4s^3$ | |
| 11. $2(a + b) + 4m(a + b)$ | 12. $4(y - 2)^2 + 3(y - 2)$ | |
| 13. $(5r - 6)(r + 3) - (2r - 1)(r + 3)$ | 14. $(3z + 2)(z + 4) - (z + 6)(z + 4)$ | |
| 15. $2(m - 1) - 3(m - 1)^2 + 2(m - 1)^3$ | 16. $5(a + 3)^3 - 2(a + 3) + (a + 3)^2$ | |

Factor each polynomial by grouping. See Example 2.

- | | |
|--------------------------------|---------------------------------|
| 17. $6st + 9t - 10s - 15$ | 18. $10ab - 6b + 35a - 21$ |
| 19. $2m^4 + 6 - am^4 - 3a$ | 20. $15 - 5m^2 - 3r^2 + m^2r^2$ |
| 21. $20z^2 - 8x + 5pz^2 - 2px$ | |

22. **Concept Check** Layla factored $16a^2 - 40a - 6a + 15$ by grouping and obtained $(8a - 3)(2a - 5)$. Jamal factored the same polynomial and gave an answer of $(3 - 8a)(5 - 2a)$. Which answer is correct?

Factor each trinomial. See Examples 3 and 4.

- | | | |
|----------------------------------|------------------------------------|---------------------------------|
| 23. $6a^2 - 11a + 4$ | 24. $8h^2 - 2h - 21$ | 25. $3m^2 + 14m + 8$ |
| 26. $9y^2 - 18y + 8$ | 27. $6k^2 + 5kp - 6p^2$ | 28. $14m^2 + 11mr - 15r^2$ |
| 29. $5a^2 - 7ab - 6b^2$ | 30. $12s^2 + 11st - 5t^2$ | 31. $9x^2 - 6x^3 + x^4$ |
| 32. $30a^2 + am - m^2$ | 33. $24a^4 + 10a^3b - 4a^2b^2$ | 34. $18x^5 + 15x^4z - 75x^3z^2$ |
| 35. $9m^2 - 12m + 4$ | 36. $16p^2 - 40p + 25$ | 37. $32a^2 + 48ab + 18b^2$ |
| 38. $20p^2 - 100pq + 125q^2$ | 39. $4x^2y^2 + 28xy + 49$ | 40. $9m^2n^2 + 12mn + 4$ |
| 41. $(a - 3b)^2 - 6(a - 3b) + 9$ | 42. $(2p + q)^2 - 10(2p + q) + 25$ | |

$$\begin{aligned} &5(2p - 5q)^2 \quad 39. (2xy + 7)^2 \\ &(3mn + 2)^2 \\ &(a - 3b - 3)^2 \\ &(2p + q - 5)^2 \quad 43. \text{ (a) B} \\ &\text{C (c) A (d) D} \quad 44. \text{ (a) B} \\ &\text{C (c) A} \\ &(3a + 4)(3a - 4) \\ &(4q + 5)(4q - 5) \\ &(5s^2 + 3t)(5s^2 - 3t) \\ &9(2z + 3y^2)(2z - 3y^2) \\ &(a + b + 4)(a + b - 4) \\ &(p - 2q + 10)(p - 2q - \end{aligned}$$

43. **Concept Check** Match each polynomial in Column I with its factored form in Column II.

- | I | II |
|--------------------------|-----------------------|
| (a) $x^2 + 10xy + 25y^2$ | A. $(x + 5y)(x - 5y)$ |
| (b) $x^2 - 10xy + 25y^2$ | B. $(x + 5y)^2$ |
| (c) $x^2 - 25y^2$ | C. $(x - 5y)^2$ |
| (d) $25y^2 - x^2$ | D. $(5y + x)(5y - x)$ |

44. **Concept Check** Match each polynomial in Column I with its factored form in Column II.

- | I | II |
|-----------------|------------------------------|
| (a) $8x^3 - 27$ | A. $(3 - 2x)(9 + 6x + 4x^2)$ |
| (b) $8x^3 + 27$ | B. $(2x - 3)(4x^2 + 6x + 9)$ |
| (c) $27 - 8x^3$ | C. $(2x + 3)(4x^2 - 6x + 9)$ |

$$\begin{aligned} &p^2 + 25)(p + 5)(p - 5) \\ &m^2 + 9)(m + 3)(m - 3) \\ &(2 - a)(4 + 2a + a^2) \\ &r + 3)(r^2 - 3r + 9) \\ &5x - 3)(25x^2 + 15x + 9) \\ &2m - 3n)(4m^2 + 6mn + \end{aligned}$$

Factor each polynomial. See Examples 5 and 6.

$$\begin{aligned} &3y^3 + 5z^2)(9y^6 - 15y^3z^2 + \\ &7(z + 3y)(z^2 - 3zy + 9y^2) \\ &(r^2 + 18r + 108) \\ &(b^2 + 9b + 27) \\ &3 - m - 2n)(9 + 3m + \\ &m^2 + 4mn + 4n^2) \\ &5 - 4a + b)(25 + 20a - \\ &16a^2 - 8ab + b^2) \quad 63. \text{ B} \end{aligned}$$

- | | | |
|----------------------|-----------------------|------------------------|
| 45. $9a^2 - 16$ | 46. $16q^2 - 25$ | 47. $25s^4 - 9t^2$ |
| 48. $36z^2 - 81y^4$ | 49. $(a + b)^2 - 16$ | 50. $(p - 2q)^2 - 100$ |
| 51. $p^4 - 625$ | 52. $m^4 - 81$ | 53. $8 - a^3$ |
| 54. $r^3 + 27$ | 55. $125x^3 - 27$ | 56. $8m^3 - 27n^3$ |
| 57. $27y^9 + 125z^6$ | 58. $27z^3 + 729y^3$ | 59. $(r + 6)^3 - 216$ |
| 60. $(b + 3)^3 - 27$ | 61. $27 - (m + 2n)^3$ | 62. $125 - (4a - b)^3$ |

63. **Concept Check** Which of the following is the correct complete factorization of $x^4 - 1$?

- | | |
|-------------------------|------------------------------|
| A. $(x^2 - 1)(x^2 + 1)$ | B. $(x^2 + 1)(x + 1)(x - 1)$ |
| C. $(x^2 - 1)^2$ | D. $(x - 1)^2(x + 1)^2$ |

64. **Concept Check** Which of the following is the correct factorization of $x^3 + 8$?

- | | |
|----------------------------|----------------------------|
| A. $(x + 2)^3$ | B. $(x + 2)(x^2 + 2x + 4)$ |
| C. $(x + 2)(x^2 - 2x + 4)$ | D. $(x + 2)(x^2 - 4x + 4)$ |

Relating Concepts

For individual or collaborative investigation
(Exercises 65–70)

The polynomial $x^6 - 1$ can be considered either a difference of squares or a difference of cubes. Work Exercises 65–70 in order, to connect the results obtained when two different methods of factoring are used.

65. Factor $x^6 - 1$ by first factoring as the difference of squares, and then factor further by using the patterns for the sum of cubes and the difference of cubes.
66. Factor $x^6 - 1$ by first factoring as the difference of cubes, and then factor further by using the pattern for the difference of squares.
67. Compare your answers in Exercises 65 and 66. Based on these results, what is the factorization of $x^4 + x^2 + 1$?
68. The polynomial $x^4 + x^2 + 1$ cannot be factored using the methods described in this section. However, there is a technique that allows us to factor it, as shown on the next page.