

## Numerical Linear Algebra

### Final Exam Fall 2004

1. Let  $A = QR$  be the factorization of a  $A$  into the product of a unitary matrix and a triangular matrix. Suppose that the columns of  $A$  are linearly independent. Show that  $|r_{kk}|$  is the distance from the  $k$ -th column of  $A$  to the linear space spanned by the first  $k - 1$  columns of  $A$ .
2. Let  $A \in \mathbb{C}^{n \times n}$  and  $b \in \mathbb{C}^n$  be arbitrary. Show that any  $x \in \mathbb{C}^n$  is equal to  $p(A)b$  for some polynomial  $p$  of degree  $\leq n - 1$ .