

Let $\alpha > 0$. Prove that $\log x \leq x^\alpha$ for x large. Prove that there exists a constant C_α such that $\log x \leq C_\alpha x^\alpha$ for all $x \in [1, \infty)$, $C_\alpha \rightarrow \infty$ as $\alpha \rightarrow 0+$, and $C_\alpha \rightarrow 0$ as $\alpha \rightarrow \infty$

Please justify all steps and be rigorous because it is an analysis problem. (Note: The problem falls under the chapter on Differentiability on \mathbb{R} in the section entitled The Mean Value Theorem, and the hint says: Find the maximum of $f(x) = \log x/x^\alpha$ for $x \in [1, \infty)$)