

Let

$$f(x) = \begin{cases} e^{-1/x^2} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Show that the  $n$ th derivative of  $f(x)$  exists for all  $n \in \mathbb{N}$ . Please justify all steps and be rigorous because it is an analysis problem. (Note: The problem falls under the chapter on Differentiability on  $\mathbb{R}$  in the section entitled The Mean Value Theorem.)