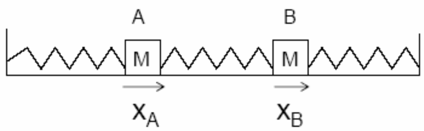
2. Two equal masses on an effectively frictionless horizontal surface are held between rigid supports by three identical springs, as shown. The displacements from equilibrium along the line of the springs are described by the coordinates x_A and x_B.



If either of the masses is clamped (held fixed), the period T for one complete vibration of the other is 3 seconds.

a) If both masses are free to vibrate, calculate the *periods* T_1 and T_2 corresponding to the normal mode frequencies, ω_1 and ω_2, for this system.

Now consider the following situation: At t = 0 mass A is in its normal resting condition and mass B is pulled aside (along the direction of the springs) a distance of B_0 = 5cm. The masses are then released from rest at this instant.

b) Show that the subsequent displacements of the two masses obey the following equations as a function of time:

$$x_A = B_0\sin[(\omega_1 + \omega_2)t/2]\sin[(\omega_2 - \omega_1)t/2]$$

$$x_B = B_0\cos[(\omega_1 + \omega_2)t/2]\cos[(\omega_2 - \omega_1)t/2]$$

where ω_1 and ω_2 are the normal mode frequencies of the system.

c) Make a sketch (using a graphics program if you have one) of the two functions $x_A(t)$ and $x_B(t)$ and calculate the length of time (in seconds) that characterizes the transfer of motion from B to A and back again.

[Hint: use the equations we derived in class for this system, putting in the appropriate initial conditions for position and speed of each mass]

d) After one cycle (motion transferred from B to A and back to B), is the situation at t = 0 exactly reproduced? Explain.

(Please show each step of your solution and FINAL ANSWERS. Thank you.)