

1. **Stadium Wave.** During the rather uneventful first half of the Michigan – Penn State game on October 15, many Michigan fans amused themselves by performing “The Wave”.

In one version of this ritual, a person will begin to raise their arms, the person standing next to them will then replicate this motion, and then their neighbor, and so on, producing a wave-like swell which travels around the stadium. We will estimate the frequency, wavelength, and propagation speed, of such a wave.

Such a traveling wave obeys the expression:

$$y = y_0 \cos(kx - \omega t)$$

where  $y$  is the displacement (in this case the height of the person’s finger tips relative to the horizontal),  $y_0$  is the maximum displacement (the **amplitude** of the wave),  $x$  is the distance traveled by the wave disturbance,  $\omega$  is the **angular frequency**, and  $k = 2\pi/\lambda$  (where  $\lambda$  is the **wavelength**).  $k$  is known as the **wave number**, or wave vector.

a) First estimate  $\omega$ , the angular frequency. The physical pendulum is useful here, in other words the natural oscillation period of the arm swinging free around the shoulder joint. In this case  $\omega = \sqrt{mgd/I}$  where  $d$  is the distance from the center of mass of the arm to the pivot point, and  $I$  is the moment of inertia of the arm. [Remember how we used this earlier to estimate the natural walking speed]. Now calculate the period of the oscillation,  $T = 2\pi/\omega$ .

b) Now estimate the wavelength. Hint: assume people are spaced about 1m apart in a stadium. This is not the wavelength, it is much longer than 1m. When you see your neighbor start to raise her arms, you will follow after a little delay. What determines this delay time (and how long approximately is the delay time?). How many people will be able to pass on the disturbance to their neighbor during one complete period of the oscillation. In this way you can estimate the wavelength.

c) Using the values estimated in parts a) and b), make an estimate of the propagation speed (phase velocity) of the wave.

**(Please show each step of your solution and FINAL ANSWERS. Thank you.)**