

11. Consider an electric circuit like that in Example 5 of Section 8.6. Assume the electromotive force is an alternating current generator which produces a voltage $V(t) = E \sin \omega t$, where E and ω are positive constants (ω is the Greek letter *omega*). If $I(0) = 0$, prove that the current has the form

$$I(t) = \frac{E}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t - \alpha) + \frac{E\omega L}{R^2 + \omega^2 L^2} e^{-Rt/L}$$

where α depends only on ω , L , and R . **Show that $\alpha = 0$ when $L = 0$.**

“SECTION 8.6

EXAMPLE 5. Electric Circuits. Figure 8.2(a), page 318, shows an electric circuit which has an electromotive force, a resistor, and an inductor connected in series. The electromotive force produces a voltage which causes an electric current to flow in the circuit. If the reader is not familiar with electric circuits, he should not be concerned. For our purposes, all we need to know about the circuit is that the voltage, denoted by $V(t)$, and the current, denoted by $I(t)$, are functions of time t related by a differential equation of the form

$$LI'(t) + RI(t) = V(t).$$

Here L and R are assumed to be positive constants. They are called respectively, the inductance and resistance of the circuit.

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The usual type of question concerning such circuits is this: If a given voltage $V(t)$ is impressed on the circuit, what is the resulting current $I(t)$? Since we are dealing with a first-order linear differential equation, the solution is a routine matter. If $I(0)$ denotes the initial current at time $t = 0$, the equation has the solution

$$I(t) = I(0)e^{-Rt/L} + e^{-Rt/L} \int_0^t \frac{V(x)}{L} e^{Rx/L} dx”$$

This is the relevant section from Single-Variable Calculus, Volume I, by Tom M. Apostol.