

(a) Let f be the function

$$f(x) = \begin{cases} \frac{1}{4}, & x = 0, \\ x, & 0 < x < 1, \\ \frac{3}{4}, & x = 1. \end{cases}$$

(ii) Using the standard partition P_n of $[0, 1]$, where $n \geq 4$, show that

$$L(f, P_n) = \frac{2n^2 - 3n + 4}{4n^2}$$

and

$$U(f, P_n) = \frac{2n^2 + 3n - 4}{4n^2}$$

(iii) Deduce that f is integrable on $[0, 1]$, and evaluate

$$\int_0^1 f.$$