Help 1.

Prove that

$$\frac{\sqrt{2}\pi}{2} \leq \int_0^{\frac{\pi}{2}} (1+x)\sqrt{2+\sin x}\, dx \leq \frac{\pi\sqrt{3}}{4}(2+\pi).$$

Help 2

Use Stirling's Formula to determine a number λ such that

$$\binom{4n}{2n} \sim \frac{\lambda}{\sqrt{n}}(16)^n \text{ as } n \longrightarrow \infty.$$

[Recall that $\binom{p}{q} = \frac{p!}{q!(p-q)!}$.]