Theorem 8.3. Assume P and Q are continuous on an open interval I. Choose any point a in I and let b be any real number. Then there is one and only one function y = f(x) which satisfies the initial value problem

y' + P(x)y = Q(x), with f(a) = b

on the interval I. This function is given by the formula

$$f(x) = be^{-A(x)} + e^{-A(x)} \int Q(t)e^{A(t)} dt$$

Where $A(x) = \int_a^x P(t)dt$

Please provide solutions to the following three problems (using the above Theorem):

3

$y' + y\tan x = \sin 2x$ on $(-\frac{1}{2}\Pi, \frac{1}{2}\Pi)$ with y = 2 when x = 0.

5

$\frac{dx}{dt} + x = e^{2t}$ on (-∞, ∞), with x = 1 when t = 0.

11.

Prove that there is exactly one function f, continuous on the positive real axis, such that

$$f(x) = 1 + \frac{1}{x}\int_1^x f(t)dt$$

for all x > 0 and find this function.