

### 3. (Rootfinding and Optimization)

- Suppose that  $f$  is differentiable on  $[a, b]$ . Discuss how you might use a rootfinding method to identify a local extremum of  $f$  inside  $[a, b]$ .
- Let  $f(x) = \log x - \cos x$ . Prove that  $f$  has a unique maximum in the interval  $[3, 4]$ . (NB:  $\log$  means natural logarithm.)
- Approximate this local maximum using six iterations of the enclosure methods (Bisection and False Position) with starting interval  $[3, 4]$ .
- Approximate this local maximum using six iterations of the two fixed-point methods (Secant and Newton). For Newton's Method, use  $p_0 = 3$ . For the Secant Method, use  $p_0 = 4$  and  $p_1 = 3$ .
- What is your best estimate for  $p$ , the location of the maximum?
- Provide the following two tables, comparing the four algorithms. The headings for the two tables should be the following.

	Approximation $p_n$			
Iteration $n$	Bisection	False Posn	Secant	Newton

	Absolute Error $ p_n - p $			
Iteration $n$	Bisection	False Posn	Secant	Newton

- Plot the absolute error for all four methods on the same graph.
- What happens if you attempt to approximate the maximum by starting Newton's Method with  $p_0 = 5$ ?