

1. (Finite Differences) Let $f(x) = \cos(x+2)$. Compute $f'(0)$ using the difference quotients given below and step-size $h = 2^{-n}$, $n = 1, \dots, 5$.

$$D_+f = \left(f(x+h) - f(x) \right) / h$$

$$D_0f = \left(f(x+h) - f(x-h) \right) / 2h$$

For each difference formula, make a table with the following information.

column 1: h

column 2: Df

column 3: $f'(0) - Df$

column 4: $(f'(0) - Df)/h$

column 5: $(f'(0) - Df)/h^2$

column 6: $(f'(0) - Df)/h^3$

Discuss the results, especially the rate of convergence.