We have the expression for g in ~~cylindrical~~ spherical polar coordinates:

$$g(r,\theta,\phi) = r\cos\theta + r\sin\theta\sin\phi - \frac{1}{2}r^2\sin^2\theta\sin(2\phi)$$

The gradient function in spherical polar coordinates of a scalar field g is:

$$\underline{\text{grad}}\, g = \underline{e}_r \frac{\partial g}{\partial r} + \underline{e}_\theta \frac{1}{r}\frac{\partial g}{\partial \theta} + \underline{e}_\phi \frac{1}{r\sin\theta}\frac{\partial g}{\partial \phi}$$

where $\underline{e}_r$, $\underline{e}_\theta$ and $\underline{e}_\phi$ are unit vectors in the r, θ- and ϕ- directions.

Thus, we calculate the partial derivatives as:

$$\frac{\partial g}{\partial r} = \cos(\theta) + \sin(\theta)\sin(\phi) - r\sin(2\phi) + r\sin(2\phi)\cos^2(\theta)$$

$$\frac{\partial g}{\partial \theta} = -r\sin(\theta) + r\cos(\theta)\sin(\phi) - r^2\sin(\theta)\sin(2\phi)\cos(\theta)$$

$$\frac{\partial g}{\partial \phi} = r\sin(\theta)\cos(\phi) - r^2\cos(2\phi) + r^2\cos(2\phi)\cos^2(\theta)$$

Substituting these values gives:

$$\begin{aligned}\underline{\text{grad}}\, g = &\left(\cos\theta + \sin\theta\sin\phi - r\sin(2\phi) + r\sin(2\phi)\cos^2\theta\right)\underline{e}_r \\ &+ 1/r\left(-r\sin\theta + r\cos\theta\sin\phi - r^2\sin\theta\sin(2\phi)\cos\theta\right)\underline{e}_{\theta} \\ &+ 1/r\sin\theta\left(r\sin\theta\cos\phi - r^2\cos(2\phi) + r^2\cos(2\phi)\cos^2\theta\right)\underline{e}_\phi\end{aligned}$$

$$=$$