1. Let *E*1, *E*2, *E*3, and *E*4 denote, respectively, the edges *ab, bc, cd,* and *da* of the square in the figure below. Write the permutation induced on {*E*1, *E*2, *E*3, *E*4} by each isometry (symmetry) of the square. [Example: ρ*H* ↦ (*E*1*E*3).] Does the symmetry group of the square act faithfully on {*E*1, *E*2, *E*3, *E*4}



**2)**

The group *G* = 〈(1 2 3 4)(5 6)〉 is of order 4 and acts on {1, 2, 3, 4, 5, 6}.

1. Determine Orb(*k*) for 1 ≤ *k* ≤ 6.
2. Determine *Gk* for 1 ≤ *k* ≤ 6.
3. Use parts (a) and (b) to verify that |Orb(*k*)| = |*G*|/|*Gk*| for 1 ≤ *k* ≤ 6.

**3)** Let *S* denote the collection of all subgroups of a finite group *G*. For *a* ϵ *G* and *H* ϵ*S*, let *πa*(*H*) = *aHa*–1.

1. Verify that with this definition *G* acts on *S*. (Each subgroup *aHa*–1 is called a *conjugate* of *H*.)
2. For *G* = *S*3, determine Orb(〈(1 2)〉).
3. For *G* = *S*3, determine *G*〈(1 2)〉.
4. Use parts (b) and (c) to verify that



1. For *H* ϵ *S*, the *normalizer* of *H* in *G* is defined by *NG*(*H*) = {*a* ϵ *G* :*aHa*–1 =*H*}. Using results from this section, explain why *NG*(*H*) is a subgroup of*G*, and the number of conjugates of *H* is [*G* : *NG*(*H*)].

**4)** Prove that if a finite group *G* contains a subgroup *H* ≠ *G* such that |G| ∤ [*G* :*H*]!, then *H* contains a nontrivial normal subgroup of *G. (note use* Lagrange’s Theorem, and facts about homomorphisms.)

***5)*** Use Burnside’s Counting Theorem to compute the number of orbits for the group 〈(1 2 3 4)(5 6)〉 acting on {1, 2, 3, 4, 5, 6}. What are the orbits?

6) Consider the problem of painting the edges of a square so that one is red, one is white, one is blue, and one is yellow.

1. In how many distinguishable ways can this be done if the edges of the square are distinguishable?
2. Repeat (a), except count different ways as being indistinguishable if one can be obtained from the other by rotation of the square in the plane.
3. Repeat (b), except permit reflections through lines as well as rotations in the plane.

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