4. In each case, solve $\nabla^2\psi = 0$ for $\psi(x, y)$ for the specified region D and the specified boundary conditions, together with the condition that ψ be bounded on D. If useful, you may use the known solution of the Dirichlet problem for the half plane [(12) in Section 20.4], as we did in Example 2.

(a) Let D be the strip $0 < y < \pi$, $-\infty < x < \infty$. Let $\psi = 0$ everywhere on the boundary except on $y = 0$ $(-\infty < x < 0)$, where $\psi = 20$. Also, evaluate $\psi(0, \pi/2)$. HINT: Use entry 6 in Appendix F.

(b) The same as part (a) but with $0 < y < 5$ instead of $0 < y < \pi$. HINT: Modify the mapping given in entry 6 slightly, so that D' is once again the upper half plane.

(c) Let D be the region $0 < x < \infty$, $0 < y < \infty$. Let $\psi = 0$ everywhere on the boundary except on $x = 0$ $(2 < y < \infty)$, where $\psi = 35$. Also, evaluate $\psi(2, 2)$. HINT: Use entry 5 in Appendix F.

(d) The same as part (c), but with different boundary conditions. This time, let $\psi = 0$ everywhere on the boundary except on $x = 0$ $(0 < y < 2)$, where $\psi = 35$.

(e) Let D be the 45° wedge between the positive x axis and the line $y = x$. Let $\psi = 0$ everywhere on the boundary except on the x axis from $x = 5$ to $x = \infty$, where $\psi = 200$. Also evaluate $\psi(5, 1)$. HINT: Use entry 5 in Appendix F.

(f) The same as part (e) but with different boundary conditions. This time let $\psi = 10$ everywhere on the boundary except on the x axis from $x = 0$ to $x = 10$, where $\psi = 0$.

(g) Let D be the semi-infinite strip $0 < x < 1$, $0 < y < \infty$. Let $\psi = 0$ on $x = 0$ and on $y = 0$, and let $\psi = 400$ on $x = 1$. Also, evaluate $\psi(0.5, 0.1)$. HINT: Use entry 10 in Appendix F.

(h) Let D be the semi-infinite strip $0 < x < 4$, $0 < y < \infty$. Let $\psi = 0$ on $y = 0$, and let $\psi = 200$ on $x = 0$ and $x = 4$. Also, evaluate $\psi(2, 2)$, $\psi(2, 4)$, $\psi(2, 6)$, $\psi(2, 8)$, $\psi(2, 10)$, and $\psi(2, 20)$. HINT: You can use entry 10 in Appendix F, but will need to modify it slightly.

(i) Let D be the semi-infinite strip $-2 < x < 2$, $0 < y < \infty$. Let $\psi = 0$ on $x = -2$, and on $y = 0$ $(-2 < x < 0)$, and let $\psi = 50$ on $x = 2$ and $y = 0$ $(0 < x < 2)$. Also, evaluate $\psi(0, 1)$ and $\psi(0, 30)$. HINT: With a slight modification you can use entry 12 in Appendix F.

✓(j) Let D be the region in the right half plane, between the lines $y = \pm x$ and the curve $x^2 - y^2 = 4$. Let $\psi = 30$ on $y = \pm x$ and let $\psi = 20$ on $x^2 - y^2 = 4$. Also, evaluate $\psi(1, 0)$ and $\psi(0.6, -0.4)$. HINT: Use the mapping $w(z) = z^2$.

(The problems are from Conformal Mapping. Please solve for only part *i*.)

(12) in Section 20.4:

Thus,

$$\hat{u}(\omega, y) = \hat{f}(\omega)e^{-|\omega|y}.$$

If we use entry 1 in Appendix D, along with the Fourier convolution property (entry 21), we obtain the final result

$$u(x, y) = f(x) * \frac{y}{\pi}\frac{1}{x^2 + y^2}, \tag{11}$$

or

$$\boxed{\begin{aligned} u(x, y) &= \frac{y}{\pi}\int_{-\infty}^{\infty} \frac{f(\xi)}{(x-\xi)^2 + y^2}\, d\xi \\ &\equiv \int_{-\infty}^{\infty} P(\xi - x, y) f(\xi)\, d\xi; \end{aligned}} \tag{12}$$

The Entry 12:

12. $w = \sin z$