3 (a) Find the Green's function $G(t,z)$ for the one point boundary value problem (equation ~~4.1~~) 3.1

$$\frac{d^2y}{dt^2}+5\frac{dy}{dt}+6y=f(t) \qquad y(0)=y'(0)=0$$

equation 3.1

and hence show the generic solution to equation 3.1 is

$$y(t)=e^{-2t}\int_0^t e^{2z}f(z)dz-e^{-3t}\int_0^t e^{3z}f(z)dz$$

[12 marks]

(b) Hence find the solution to *equation 3.1* if $f(t)=\sinh t$

[5 marks]

(c) Show that the eigenvalues λ_n and normalised eigenfunctions $y_n(x)$ for the eigenvalue equation given in equation 3c

$$\frac{d^2y}{dx^2}+4y=\lambda y \qquad y'(0)=y(1)=0 \qquad \text{equation 3c}$$

where λ is an undetermined constant, are given by –

$$\lambda_n=\left(4-\frac{(2n-1)^2\pi^2}{4}\right) \qquad y_n(x)=\sqrt{2}\cos\left(\frac{2n-1}{2}\right)\pi x$$

where *n* = 1,2,3 ………

[10 marks]

(d) Use the results of part (c) to <u>write down</u> the Green's function for *equation 3d*, explaining why it is appropriate to do so.

$$\frac{d^2y}{dx^2}+4y=f(x) \qquad y'(0)=y(1)=0 \qquad \textit{Equation 3d}$$

[3 marks]