2. Suppose you have a two-period model OLG in discrete time. Lt agents are born in time t, where Lt = (1+n)tL0. Normalize L0 = 1 and let n > 0. Preferences of a young agent born in time t are time separable:

u(c1t,c2t+1) = 2(c1t)0.5 + 2(c2t+1)0.5,

where c1t denotes consumption in period t; c2t+1 denotes consumption in period t+1 of an old agent born in t. The initial old agents want to consume as much as possible. Each young agent is endowed with y units of the consumption good. The old have no endowment whatsoever. There is a storage technology that allows to convert one unit of period t goods into 1+r units of period t+1 goods. There is a social security system that is “pay-as-you-go”. In each period t the government taxes the young and uses the receipts to make transfers to the old. We consider a per capita tax on the young that is constant over time, i.e. τt = τ for all t = 0, 1, 2, …

i) What is the government budget constraint in period t? Write it both in aggregate and in per capita terms.

ii) Solve for the competitive equilibrium consumption levels. Also find the savings of the representative young agent st.

iii) What is the effect of an increase in τ on savings of the representative young agent?

iv) What is the Pareto optimal tax rate τ if n < r? Explain why.