Table 17.4 Data for three-dimensional conformal coordinate transformation

Point	X	Y	Z	$x \pm S_x$	$y \pm S_y$	$z \pm S_z$
1	10037.81	5262.09	772.04	1094.883 ± 0.007	820.085 ± 0.008	109.821 ± 0.005
2	10956.68	5128.17	783.00	503.891 ± 0.011	1598.698 ± 0.008	117.685 ± 0.009
3	8780.08	4840.29	782.62	2349.343 ± 0.006	207.658 ± 0.005	151.387 ± 0.007
4	10185.80	4700.21	851.32	1395.320 ± 0.005	1348.853 ± 0.008	215.261 ± 0.009
5				265.346 ± 0.005	1003.470 ± 0.007	78.609 ± 0.003
6				784.081 ± 0.006	512.683 ± 0.008	139.551 ± 0.008

```
3D Coordinate Transformation of File: EX17-4.DAT
Iteration 1
-----------------------------------------------

J MATRIX                                                    K MATRIX
-------------------------------------------------------     --------
    0.000  102.452  1284.788 1.000 0.000 0.000  -206.164      -0.000
  -51.103   -7.815  -195.197 0.000 1.000 0.000 -1355.718       0.000
-1287.912  195.697     4.553 0.000 0.000 1.000    53.794       0.000
    0.000  118.747  1418.158 1.000 0.000 0.000   761.082      -0.000
  -62.063   28.850   723.004 0.000 1.000 0.000 -1496.689      -0.000
-1421.832 -722.441    42.501 0.000 0.000 1.000    65.331      -0.000
    0.000  129.863  1706.020 1.000 0.000 0.000 -1530.174       0.060
  -61.683  -58.003 -1451.826 0.000 1.000 0.000 -1799.945       0.209
-1709.922 1452.485   -41.580 0.000 0.000 1.000    64.931       0.000
    0.000  204.044  1842.981 1.000 0.000 0.000   -50.417       0.033
 -130.341   -1.911   -46.604 0.000 1.000 0.000 -1947.124      -0.053
-1849.740   47.857    15.851 0.000 0.000 1.000   137.203       0.043

X MATRIX
------------
-0.0000347107
-0.0000103312
-0.0001056763
 0.1953458986
-0.0209088384
-0.0400969773
-0.0000257795
Iteration 2
J MATRIX                                                    K MATRIX
-------------------------------------------------------     --------
    0.000  102.446  1284.733 1.000 0.000 0.000  -206.308      -0.064
  -51.144   -7.813  -195.328 0.000 1.000 0.000 -1355.694      -0.037
-1287.855  195.829     4.567 0.000 0.000 1.000    53.838      -0.001
    0.000  118.750  1418.196 1.000 0.000 0.000   760.923      -0.025
  -62.114   28.818   722.835 0.000 1.000 0.000 -1496.767       0.057
-1421.868 -722.272    42.484 0.000 0.000 1.000    65.386      -0.011
    0.000  129.843  1705.820 1.000 0.000 0.000 -1530.365       0.007
  -61.730  -57.958 -1451.969 0.000 1.000 0.000 -1799.781       0.028
-1709.719 1452.630   -41.520 0.000 0.000 1.000    64.982      -0.007
    0.000  204.036  1842.926 1.000 0.000 0.000   -50.624       0.033
 -130.399   -1.917   -46.797 0.000 1.000 0.000 -1947.114      -0.091
-1849.680   48.053    15.864 0.000 0.000 1.000   137.268       0.024
```

X MATRIX

```
 0.0000000007
 0.0000000046
-0.0000000033
-0.0000000588
-0.0000008524
 0.0000030138
 0.0000000053
```

Measured Points

Point	x	y	z	Sx	Sy	Sz
1	1094.883	820.085	109.821	0.007	0.008	0.005
2	503.891	1598.698	117.685	0.011	0.008	0.009
3	2349.343	207.658	151.387	0.006	0.005	0.007
4	1395.320	1348.853	215.261	0.005	0.008	0.009

Control Points

Point	X	VX	Y	VY	Z	VZ
1	10037.810	0.064	5262.090	0.037	772.040	0.001
2	10956.680	0.025	5128.170	-0.057	783.000	0.011
3	8780.080	-0.007	4840.290	-0.028	782.620	0.007
4	10185.800	-0.033	4700.210	0.091	851.320	-0.024

Transformation Coefficients

```
Scale =         0.94996 +/- 0.00004
Omega =    2° 17' 05.3" +/- 0° 00' 30.1"
  Phi =   -0° 33' 02.8" +/- 0° 00' 09.7"
Kappa =  224° 32' 10.9" +/- 0° 00' 06.9"
   Tx =       10233.858 +/-      0.065
   Ty =        6549.981 +/-      0.071
   Tz =         720.897 +/-      0.213
```

```
Reference Standard Deviation: 8.663
Degrees of Freedom: 5
Iterations: 2
```

Transformed Coordinates

Point	X	Sx	Y	Sy	Z	Sz
5	10722.020	0.073	5691.221	0.080	766.068	0.248
6	10043.246	0.072	5675.898	0.080	816.867	0.248

Dox MATRIX

```
 0.0000000003  0.0000000000  0.0000000000 -0.0000000068 0.0000000210  0.0000004081 -0.0000000000
 0.0000000000  0.0000000000 -0.0000000000 -0.0000000039 0.0000000025  0.0000000487 -0.0000000000
-0.0000000000 -0.0000000000  0.0000000000 -0.0000000247 0.0000000064  0.0000000008 -0.0000000000
-0.0000000068 -0.0000000039 -0.0000000247  0.0000561837 0.0000020823 -0.0000095524  0.0000000101
 0.0000000210  0.0000000025  0.0000000064  0.0000020823 0.0000669290  0.0000283700  0.0000000296
 0.0000004081  0.0000000487  0.0000000008 -0.0000095524 0.0000283700  0.0006032234 -0.0000000016
-0.0000000000 -0.0000000000 -0.0000000000  0.0000000101 0.0000000296 -0.0000000016  0.0000000000
```

Note that in this adjustment, with four control points available having X, Y, and Z coordinates, 12 equations could be written, three for each point. With seven unknown parameters, this gave $12 - 7 = 5$ degrees of freedom in the solution.

17.8 STATISTICALLY VALID PARAMETERS

Besides the coordinate transformations described in preceding sections, it is possible to develop numerous others. For example, polynomial equations of various degrees could be used to transform data. As additional terms are added to a polynomial, the resulting equation will force better fits on any given data set. However, caution should be exercised when doing this since the resulting transformation parameters may not be statistically significant.

As an example, when using a two-dimensional conformal coordinate transformation with a data set having four control points, nonzero residuals would be expected. However, if a projective transformation were used, this data set would yield a unique solution and thus the residuals would be zero. Is the projective a more appropriate transformation for this data set? Is this truly a better fit? Guidance in the answers to these questions can be obtained by checking the statistical validity of the parameters.

The adjusted parameters divided by their standard deviations represent a t statistic with v degrees of freedom. If a parameter is to be judged as statistically different from zero, and thus significant, the computed t value (the test statistic) must be greater than $t_{\alpha/2,v}$. Simply stated, the test statistic is

$$t = \frac{|\text{parameter}|}{S} \tag{17.16}$$

For example, in the adjustment in of Example 17.2, the following computed t values are found: